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V. N. Malozemov, A. V. Plotkin

Strict polynomial separation of two sets

Suppose that there are two finite sets in $\mathbb{R}^n$:

$$A = \{a_i\}_{i=1}^{m} \quad \text{and} \quad B = \{b_j\}_{j=1}^{k}.$$

Consider the generalized polynomial

$$P(x, t) = \sum_{s=1}^{r} x[s] u_s(t), \quad t \in \mathbb{R}^n,$$

where $u_s(t)$ are continuous functions of n variables.

Definition

We say that sets A and B are strictly polynomial separable, if there exists vector $x_0 \in \mathbb{R}^r$ such that

$$P(x_0, a_i) \geqslant 1, \; i \in 1 : m, \quad P(x_0, b_j) \leqslant -1, \; j \in 1 : k.$$

Thus the separating hypersurface is defined by equation

$$P(x_0, t) = 0.$$

Finding the separating polynomial $P(x_0, t)$ reduces to linear programming problem.

We introduce the function

$$f(x) = \max\Big\{ \max_{i \in 1:m} \big[1 - P(x, a_i)\big]_+,\ \max_{j \in 1:k} \big[1 + P(x, b_j)\big]_+ \Big\},$$

where $[u]_+ = \max\{0, u\}$. It's obvious that $f(x) \geqslant 0$ for all $x \in \mathbb{R}^r$.

Lemma

Polynomial $P(x_0, t)$ strictly separates sets A and B if and only if $f(x_0) = 0$.

Consider the extremal problem

$$f(x) \to \min_{x \in \mathbb{R}^r} . \tag{1}$$

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Theorem 1

Problem (1) *is equivalent to the following linear programming problem:*

$$\begin{aligned} w &\to \min \\ P(x, a_i) + w &\geqslant 1, \quad i \in 1 : m; \\ -P(x, b_j) + w &\geqslant 1, \quad j \in 1 : k; \\ w &\geqslant 0. \end{aligned} \tag{2}$$

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Theorem 2

Let $(x_*,\ w_*)$ be a solution of problem (2). Then w_* is 0 or 1.