J Optim Theory Appl (2016) 171:351–353
DOI 10.1007/s10957-016-1019-6

PREFACE

Preface

Vasily Malozemov[1] · **Maksim Dolgopolik**[1]

Published online: 4 October 2016

Professor Vladimir Fedorovich Demyanov was born in Dnepropetrovsk on August 18, 1938. He passed away on April 18, 2014. His life was closely linked to the University of Leningrad (now St. Petersburg). In 1960, he graduated from the Department of Computational Mathematics organized by the Nobel Prize Winner L. V. Kantorovich in the early 1950s. In 1964, he was awarded his Ph.D., and he got the habilitation for full professor in 1972. Since 1988 and until the end of his days, he served as Chair of the Department of Mathematical Theory of Systems and Control. He also worked at the University of Syktyvkar (1973–1975) as President of the Faculty of Physics and Mathematics, and during the years 1983–1985, he worked at the International Institute for Applied Systems Analysis (IIASA), an international research organization located in Laxenburg, Austria (near Vienna).

Demyanov is regarded as one of the founders of nonsmooth analysis. His seminal papers on minmax and minmaxmin date back to the second half of the 1960s. In 1967–1968, during his stay with L. W. Neustadt at the University of Southern California, he authored the paper *V. F. Demyanov, Algorithms for some minimax problems, J. Computer and System Sciences* ***2****(4) 342–380 (1968)*, that set the direction of his scientific interests for decades. He achieved international recognition with the monograph *V. F. Demyanov, V. N. Malozemov, Introduction to Minimax, Nauka, Moscow (1972); English translation: John Wiley and Sons (1974); and reprinted by Dover, New York (1990).* It became one of the important texts in nondifferentiable optimization.

After the theory of minimax problems, Demyanov turned his attention to more general nonsmooth optimization problems. His desire was to extend the results obtained for minimax problems to as general a class of nonsmooth functions as possible. The

✉ Vasily Malozemov
malv@math.spbu.ru

1 Saint Petersburg State University, Saint Petersburg, Russia

inspiration for this extension came from the works of B. N. Pshenichnyi, in which a class of directionally differentiable functions whose directional derivative can be expressed through the maximum over a compact and convex set (a subdifferential) was considered a natural generalization of the class of max-functions. The main drawback of this class of functions was that it is not closed under many standard mathematical operations such as subtraction and multiplication. In order to overcome this drawback, professor Demyanov proposed what now is known as quasidifferentiable functions. Quasidifferentiable functions are those directionally differentiable functions whose directional derivatives can be represented as the difference of positively homogeneous convex functions. The class of quasidifferentiable functions forms a linear space, enjoys a simple calculus, and allows one to extend many classical results from differential calculus to a nonsmooth setting.

In the late 1980s, Demyanov became sufficiently frustrated with the direction the work on subdifferentials was taking that he intervened during a meeting presentation saying "With quasidifferential calculus, I try to get milk from the cows, but you, with your subdifferential calculus, pretend to draw the milk by crocodiles." The theory of quasidifferentiable functions or quasidifferential calculus, as Demyanov called it, was developed in conjunction with A. M. Rubinov, L. N. Polyakova and other colleagues from Leningrad State University.

Despite all its merits, the class of quasidifferentiable functions appeared to be incompatible with numerical methods, mostly due to the discontinuity of quasidifferential mapping in the nonsmooth case. Demyanov always aimed at studying approximations of nonsmooth functions that would allow one to obtain abstract mathematical results, to calculate descent directions, and to design effective numerical methods. Therefore, dissatisfied with quasidifferential calculus, he looked for a new approach to nonsmooth problems. A colleague, V. N. Malozemov, had developed a linearization method for solving minimax problems that consisted of replacing the functions under max-operation by the linear part of their Taylor expansion at a given point, thus obtaining a natural nonhomogeneous approximation of the max-function. In 1988, motivated by this approach, Demyanov introduced a new class of nonsmooth functions called codifferentiable functions and later developed the codifferential calculus. Codifferential calculus was probably the first general theory of nonsmooth functions that was based on local nonhomogeneous approximations. Namely, codifferentiable functions are those nonsmooth functions that can be locally approximated by the difference of convex functions. Furthermore, the use of nonhomogeneous approximations allowed him to construct the first continuous approximation of a nonsmooth function, since unlike all subdifferential mappings, codifferential mappings are usually continuous. The continuity of codifferential mapping as well as the simple and well-developed calculus of codifferentiable functions allowed him to design the method of codifferential descent that has been applied to a wide variety of nonsmooth optimization problems.

Professor Demyanov's work on quasidifferential and codifferential calculus culminated in the publication jointly with A. M. Rubinov of the book "*Constructive Nonsmooth Analysis*" in 1995. The term "*constructive*" characterized his approach toward nonsmooth analysis. He always wanted to develop a theory of nonsmooth functions that would allow one not only to obtain optimality conditions and study some abstract properties but also to obtain a simple calculus of nonsmooth functions. This calculus would make it easy to calculate descent directions and to design effective numerical methods that can be implemented on a computer.

After the development of codifferential calculus, professor Demyanov turned his attention back to directional derivatives and quasidifferentiable functions. A fresh look at quasidifferential calculus and some results of B. N. Pshenichnyi and A. M. Rubinov on positively homogeneous upper convex approximations led to the notion of an exhauster of a nonsmooth function in the late 1990s. The exhauster is a natural generalization of a quasidifferential that, unlike a quasidifferential, can be correctly defined for an arbitrary directionally differentiable function. This allowed Demyanov to extend his constructive approach to the class of all directionally differentiable functions. At the same time, he introduced the notion of a coexhauster of a nonsmooth function, which is to codifferential what exhauster is to quasidifferential. The theory of exhausters and coexhausters was developed by Demyanov and his students V. A. Roschina and M. Abbasov in the 2000s and early 2010s. Subsequently, separate results on quasidifferentials, codifferentials, exhausters and coexhausters were united into a general theory and extended to the infinite dimensional case by Demyanov's student M. V. Dolgopolik.

From the mid-1990s and in parallel with his work on exhausters and coexhausters, professor Demyanov worked on the theory of exact penalty functions. Since exact penalty functions are usually nonsmooth, he considered the theory of exact penalty functions as a natural application of his results from nonsmooth analysis. Apart from some general results on exact penalty functions, Demyanov found surprising applications of the theory to the calculus of variations. He managed to rederive in a very simple manner many classical results from the calculus of variations with the use of exact penalty functions and nonsmooth analysis. More importantly, he proposed a new direct numerical method, called the method of hypodifferential descent, for solving various problems of the calculus of variations. This method, based on the method of codifferential descent, was advanced by Demyanov and his student G. Sh. Tamasyan in the late 2000s–early 2010s.

His list of publications contains more than 200 items (among them 9 books in Russian, English, and other languages). He was a generous scholar, mainly to the young scientists who had the chance to work under his supervision. He was supervisor of 40 Ph.D. dissertations, and among his former students, we find A. B. Pevniy, L. N. Polyakova, and L. V. Vasiliev.

He was also Chairman of three international congresses, and many times he delivered plenary lectures at prestigious international conferences. His tireless and enthusiastic activity was highly influential for many mathematicians all over the world.