



Constructive Optimisation Workshop
in honour of Prof. Vladimir Demyanov's 75th birthday

16-17 April 2014
Federation University Australia & RMIT University

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Abstracts

Limited memory discrete gradient bundle method for nonsmooth derivative-free optimization

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Typically, practical nonsmooth optimization problems involve functions with hundreds of variables. Moreover, there are many practical problems where the computation of even one subgradient is either a difficult or an impossible task. In such cases derivative-free methods are the better (or only) choice since they do not use explicit computation of subgradients. However, these methods require a large number of function evaluations even for moderately large problems. In this article, we propose an efficient derivative-free limited memory discrete gradient bundle method for nonsmooth, possibly nonconvex optimization. The convergence of the proposed method is proved for locally Lipschitz continuous functions and the numerical experiments to be presented confirm the usability of the method especially for medium size and large-scale problems.

Douglas-Rachford Feasibility Methods For Matrix Completion Problems

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Many successful non-convex applications of the Douglas-Rachford method can be viewed as the reconstruction of a matrix, with known properties, from a subset of its entries. In this talk we discuss recent successful applications of the method to a variety of (real) matrix reconstruction problems, both convex and non-convex.

Objective-guided Decomposition for State-space Search

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We examine a heuristic decomposition approach to find feasible solutions for a class of problems with state-dependent components. In particular, we study a method to reduce the state-space search of Automated Planning problems by decomposing a problem into two partitions, where the second partition is warm-started with the solution of the first. This type of approach is usually motivated by the presence of two sub-problems that are each more easily solved by different methods. We draw on the concepts of Benders Decomposition to create an improved search space for the planner, using mixed-integer programming for the first partition. Since planning technology cannot currently return a ‘Benders cut’, our approach is uni-directional. We imitate the final Benders pass by approximating the second partition within the first. We illustrate that even using a simple approximation can have positive impact on two examples: the Travelling Purchaser Problem and a Mine Planning Problem.

On Augmented Lagrangian Duality In Integer Programming

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We consider the augmented Lagrangian dual for integer programming, and provide a primal characterization of the resulting bound. As a corollary, we obtain proof that the augmented Lagrangian is a strong dual for integer programming. We are able to show that the penalty parameter applied to the augmented Lagrangian term may be placed at a fixed, large value and still obtain strong duality for pure integer programs. Some potential avenue of application of these results are discussed.

Canonical Duality: Natural Connection Between Nonconvex Mechanics and Global Optimization

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The history for studying nonsmooth phenomena can be viewed as earlier as Henri Tresca’s work in 1864, who is the discover of the Tresca yield criterion, one of two main failure criteria used today for ductile materials. Mathematically, the Tresca yield condition leads to a nonsmooth (piecewise linear) convex set in stress space. While the other one, von Mises criterion is a smooth (ellipsoid) convex set. Tresca’s stature as an outstanding scientist was such that Gustave Eiffel put his name on number 3 in his list of 72 people making the Eiffel tower in Paris possible.

In modern mathematics, the theory of nonsmooth analysis was developed from J.J. Moreau’s pioneering work on frictional mechanics in 1963. By introducing a series of revolutionary concepts such as super-potential and subdifferential, a beautiful duality theory was developed from mathematical physics to abstract convex analysis.

Abstracts

In this talk, the speaker will first show some surprising experimental results on smooth-nonsmooth criteria and mathematical theory in nonsmooth mechanics, which may not know to the community of mathematical optimization [1]. Then some basic concepts, such as objectivity, Lagrangian, equilibrium equation, geometrical nonlinearity will be discussed along with the common misunderstandings between physics and mathematical optimization [2]. He will show how the canonical duality theory was naturally developed from these basic concepts, its philosophical foundation and solid backgrounds in both geometry and physics. By using the most simple quadratic 0-1 integer programming problem, he will show that general nonconvex/nonsmooth/discrete minimization problems can be converted to a unified canonical dual problem, which can be solved easily under certain conditions [3]. The triality theory can be used to identify NP-hard problems. In the worst case, the NP-complete 0-1 integer programming problem is identical to a continuous unconstrained nonsmooth (Lipschitzian) global optimization problem, which can be solved deterministically [4]. This talk should bring some fundamentally new insights into nonsmooth analysis and global optimization.

References:

[1] Gao, D.Y., *Duality Principles in Nonconvex Systems: Theory, Methods and Applications*. Kluwer Academic Publishers, Boston/Dordrecht/London, 2000, xviii+454pp.

[2] Gao, DY, and Wu, C. On the Triality Theory in Global Optimization, <http://arxiv.org/abs/1104.2970>

[3] Gao, D.Y. (2009). Canonical duality theory: Unified understanding and generalized solution for global optimization problems, *Computers & Chemical Engineering*, 33:1964–1972. doi:10.1016/j.compchemeng.2009.06.009

[4] Gao DY, Layne T. Watson, David R. Easterling, William I. Thacker, and Stephen C. Billups, Solving the canonical dual of box- and integer-constrained nonconvex quadratic programs via a deterministic direct search algorithm, *Optimization Methods and Software*, 28, No. 2, April 2013, 313–326

Several kinds of generalized derivatives based on radial cones and optimality conditions

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We propose several kinds of higher-order generalized derivatives of a set-valued map using radial cones in connection with the higher-order contingent derivative, the higher-order Studniarski derivative, and some higher-order epiderivatives.

About quantitative characterizations of regularity properties

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Several local regularity properties of finite collections of sets will be discussed, namely: semiregularity, subregularity and uniform regularity. Metric and dual characterizations of these properties will be provided.

Bundle Methods with Linear Subproblem

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We present a special bundle trust region method for minimizing locally Lipschitz and prox-regular functions. The paraxconvexity of such functions allows us to use the local convexification model and its convexity properties. The model is to be controlled during the iteration process such that the linearization errors are always positive. The trust region is formed by infinity norm so we have a linear subproblem in each iteration. We show that if the convexification succeeds the algorithm converges to a stationary point. Preliminary numerical experiments on academic test problems show that the algorithm is reliable and efficient.

Optimality conditions and algorithm for linear semi-infinite programming

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We present in this talk a number of recent optimality results for linear semi-infinite programming (LSIP), including optimality conditions with and without constraint qualifications and a generalization of the fundamental theorem of LP to LSIP. The latter have led to a new algorithm for LSIP, which will also be discussed.

Abstracts

Turnpike theorem for an infinite horizon optimal control problem with time delay

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An optimal control problem for systems described by a special class of nonlinear differential equations with time delay is considered. The cost functional adopted could be considered as an analogue of the terminal functional defined over an infinite time horizon. The existence of optimal solutions as well as the asymptotic stability of optimal trajectories (that is, the turnpike property) are established under some quite mild restrictions on the nonlinearities of the functions involved in the description of the problem. Such mild restrictions on the nonlinearities allowed us to apply these results to a blood cell production model.

In-pair uniform regularity of collections of sets

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We investigate qualitative and quantitative characterizations of collections of sets whose every two-set subcollection is uniformly regular. An application of this in-pair property in convergence analysis is also provided.

Calculus rules for the directed subdifferential

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We extend the exact calculus rules for the directed subdifferential of a delta-convex (DC) function to the generalised version of the directed subdifferential that does not depend on the DC structure.

Optimization-based features extraction for K-complex detection

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The K-complex is a transient electroencephalogram (EEG) waveform that contributes to sleep stages scoring. Automated detection of K-complexes in the EEG signal is an important component of sleep stage monitoring. However it is difficult due to the stochastic nature of brain signals, presence of noise, complexity and extreme size of data. We apply an optimization model, based on solving a sequence of linear least squares problems to extract key features of EEG signal. In this paper a specific procedure to detect K-complexes is developed. This procedure is based on the proposed optimization model and classification algorithms from WEKA software to enhance the accuracy in classification of extracted features. Numerical results show that this procedure is efficient for detecting K-complexes. The proposed linear least squares approach greatly reduces the number of features, as well as reduces the computational time.

Global Optimisation Based on Mixed Integer Nonlinear Programming

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By utilizing an efficient global optimisation method, the speaker will discuss how to solve diverse challenging problems involving convex/ non-convex functions and mixed variables such as integer, discrete and continuous variables. The problems are usually formulated as mixed integer nonlinear programming problem. Although many optimisation approaches have been developed to solve mixed integer nonlinear programming problems, these methods can only find an approximate solution or use constraints to reformulate the problem. In this talk a deterministic optimisation method will be presented with detail applications.

Abstracts

Fixed knots polynomial spline approximation using Quasidifferentiability

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The problem of fixed knot polynomial spline approximation in Chebyshev norm is convex. Therefore, in theory, we can solve this problem efficiently. However, there are still quite a few interesting issues that can be discussed. In this talk I will concentrate on how some classical results of polynomial approximation can be extended to the case of fixed knots polynomial splines with the use of Quasidifferentiability. Also, I am going to talk about some recent findings in the area of free knot polynomial spline approximation and multivariate approximation (using Quasidifferentiability).

Norm Convergence of Realistic Projection and Reflection Methods

Mathew Tam, The University of Newcastle

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In this talk I will discuss unified sufficient conditions for norm convergence of fundamental reflection and projection methods applied to feasibility problems having finite codimensional affine and convex cone constraints. This is joint work with Jon Borwein and Brailey Sims.

The history of polynomial spline approximation through quasidifferentiability

Julien Ugon, Federation University Australia

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In this talk, we give a quick introduction to the theory of polynomial and polynomial spline approximation through Quasidifferentiability and how it can be applied to the nonconvex problem of free knots spline approximation. Quasidifferentiability is a powerful tool of nonsmooth analysis. There are several ways to solve optimisation problems in polynomial spline approximation, however, the notion of quasidifferentiability allowed us to solve some of the problems that have been declared by the approximation research community as hard and even not possible due to its "nonlinear nature".

Efficient, parameterless Max-Min classifiers

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The problem of discriminating between two sets is very important in data mining, particularly in supervised data classification. In many data sets the classes are disjoint, but their convex hulls intersect. This makes separating the classes difficult, as the decision boundary between the classes is non-linear. Several techniques have been proposed over the last three decades to approximate decision boundaries using piecewise linear functions. Many of them try to avoid solving optimisation problems as the determination of piecewise linear boundaries is a complex optimisation task, requiring a long training time. In most cases the problem of finding such boundaries is reduced to the minimisation of the class error function, which is nondifferentiable and nonconvex. We propose two approaches that reduce the complexity of the optimisation problem. In the first approach, sets are approximated by hyperboxes to find the so-called "indeterminate" regions between sets, eliminating points outside these regions. The second approach globally and incrementally trains hyperplanes to find a near global minimizer of the classification error function, computing as fewer hyperplanes as needed for the set separation. These techniques are applied to supervised data classification problems and results of numerical experiments on real-world data sets are reported. These results demonstrate that these techniques require reasonable training time and their test set accuracy is consistently good on most data sets compared with mainstream classifiers.

Abstracts

Decomposable polytopes

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Minkowski sums and decompositions of polyhedra/polytopes occur naturally in optimization and other situations. We use graph theoretic methods to establish some geometric results about them, namely classification as decomposable or indecomposable in terms of properties of their skeletons. In particular, we have a complete list of d -dimensional decomposable polytopes with less than d squared plus one and a half d edges.

An optimal labelling problem for outerplanar graphs of maximum degree three

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I will talk about an optimal labelling problem with motivations from frequency assignment. The talk is based on a joint paper with Xiangwen Li. Given a graph G and a positive integer k , a k - $L(2,1)$ -labelling of G is an assignment of an integer from $\{0, 1, \dots, k\}$ to each vertex of G such that adjacent vertices receive integers that differ by at least two and vertices at distance two receive different integers. The minimum k such that G admits a k - $L(2,1)$ -labelling is called the λ -number of G , denoted by $\lambda(G)$. A graph is outerplanar if it can be drawn in the plan without edge crossing such that all vertices are on the boundary of a single face. Bodlaender et al. conjectured that if G is an outerplanar graph of maximum degree Δ , then $\lambda(G) \leq \Delta - 2$. Calamoneri and Petreschi proved that this conjecture is true when $\Delta \geq 8$ but false when $\Delta = 3$. Meanwhile, they proved $\lambda(G) \leq \Delta - 5$ for any outerplanar graph with $\Delta = 3$ and asked whether this bound is sharp. Answering this question, we prove that $\lambda(G) \leq \Delta - 3$ for any outerplanar graph with $\Delta = 3$. We also show that $\Delta - 3$ can be achieved by infinitely many outerplanar graphs with $\Delta = 3$.

