Test Examples for Nonlinear Programming Codes

- All Problems from the Hock-Schittkowski-Collection -

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Abstract

The test problems of the Hock and Schittkowski-collection[1] became quite popular in the past for developing and testing nonlinear programming codes. Since this first collection is out of print, we present a short review and the original scanned test problem documentation. We provide optimal solutions, program organization, and some numerical test results.

[1]Test Examples for Nonlinear Programming Codes, Willi Hock, Klaus Schittkowski, Springer, Lecture Notes in Economics and Mathematical Systems, Vol. 187

1 Introduction

A couple of years ago, the author published two test problem collections for testing nonlinear programming codes, see Hock and Schittkowski [4] and Schittkowski [9]. The problems are widely used and contained also in other test problem collections, for example in the Cute library of Bongartz et al. [1], available through the URL

`http://www.cse.clrc.ac.uk/activity/cute`

The test problem collection of Spellucci [12] is available through the ftp site

`ftp://ftp.mathematik.tu-darmstadt.de/pub/department/software/opti/`

confer also the benchmark test page maintained by Mittelmann

`http://plato.la.asu.edu/bench.html`

In addition, AMPL versions of all test problems of the two collections are available through the links

`http://www.sor.princeton.edu/~rvdb/ampl/nlmodels/hs/index.html`

and

`http://www.sor.princeton.edu/~rvdb/ampl/nlmodels/s/index.html`

see also Fourer et al. [3] for more details about AMPL. The original Fortran implementation can be downloaded from

`http://www.old.uni-bayreuth.de/departments/math/~kschittkowski/downloads.htm`

When developing a new version of a sequential quadratic programming algorithm, the test examples were investigated again and used for some numerical tests, see Schittkowski [11]. The purpose of the paper is to outline the usage of the codes and to make them available for public usage.

We consider the general optimization problem, to minimize an objective function $f(x)$ under nonlinear equality and inequality constraints,

$$x \in I\!R^n : \quad \begin{array}{ll} \min f(x) & \\ a_j^T x + \beta_j \geq 0 \ , & j = 1, \ldots, m_{11}, \\ g_j(x) \geq 0 \ , & j = m_{11}+1, \ldots, m_1, \\ a_j^T x + \beta_j = 0 \ , & j = m_1 + 1, \ldots, m_{21}, \\ g_j(x) = 0 \ , & j = m_{21}+1, \ldots, m, \\ x_l \leq x \leq x_u & \end{array} \qquad (1)$$

where x is an n-dimensional parameter vector. It is supposed that the first m_{11} inequality constraints and that the first $m_{21} - m_1$ equations are linear, whereas the remaining ones are nonlinear. To facilitate the notation, we set $g_j(x) = a_j^T x + \beta_j$ for $j = 1, \ldots, m_{11}$ and

$j = m_1 + 1, \ldots, m_{21}$. Objective function and constraints are supposed to be continuously differentiable on the whole $\mathbb{R}^n$.

The test problems have been used in the past to develop the nonlinear programming code NLPQL [8], a Fortran implementation of a sequential quadratic programming (SQP) algorithm. The design of the numerical algorithm is founded on extensive comparative numerical tests of Schittkowski [7], Schittkowski et al. [10], and Hock, Schittkowski [5]. To complete the numerical tests, an additional random test problem generator was developed for a major comparative study, see [7]. More than 100 test problems based on a finite element formulation are collected for the comparative evaluation in Schittkowski et al. [10].

All these efforts indicate the importance of a qualified set of test examples for debugging, validation, performance evaluation, and quantitative numerical comparisons with alternative codes. Although not collected in a very systematically way, the test problems represent all numerical difficulties we observe in practice, for example

1. badly scaled objective and constraint functions,
2. badly scaled variables,
3. non-smooth model functions,
4. ill-conditioned optimization problems,
5. non-regular solutions at points where the constraint qualification is not satisfied,
6. different local solutions,
7. infinitely many solutions.

Academic test problems allow either an analytical or a numerical investigation of all interesting properties, with nearly no or only limited efforts. On the other hand, nonlinear programming problems based on a *real-life* background are often too complex to serve as test problems, are often not available, are not programmed in a standard form as required for massive tests, or contain round-off and truncation errors, in particular if secondary iterative numerical algorithms are included to compute function and gradient values. The latter argument is crucial, since most non-trivial application problems generate numerical errors in the one or other form. Often gradients are only available by forward differences. However, we can use the academic test problems also in this situation, by adding randomly generated errors and by approximating derivatives numerically. A few corresponding test results are found in Schittkowski [11].

It is important that all test examples come with an optimal solution obtained by analytical or numerical experimentation investigation. Most examples are non-convex, but we hope that at least in most cases, we are able to provide a global solution. For most test problems, analytical gradients are available. However, we cannot give a guarantee that they are correct and recommend usage of numerical differentiation.

The Fortran source codes of all test problems are made available through http://www.uni-bayreuth.de/departments/math/~kschittkowski/home.htm

The usage of the subroutines is documented in Section 2 together with a detailed example. A driver program is listed that shows how a nonlinear programming code, in this case NLPQLP, would evaluate function and gradient values. The files that are provided by the author, are listed in Section 3 together with a description of the generated output. A main program that executes all 115 test examples within a loop and solves them by the code NLPQLP, see Schittkowski [11], is attached. The code contains also an evaluation of numerical results based on a decision, whether the result of a test run is considered to be a successful one or not. Some numerical tests are included in Section 4 which are helpful for comparing own implementations. An appendix contains a list of all individual results including performance data, i.e. number of function calls, number of iterations, errors in objective function, and constraint violations.

Since the first test problem collection [4] is out of print, an appendix is attached which contains a detailed documentation of all problems in the original form.

2 Usage of the Fortran Subroutines

A test problem is set up by

CALL TPno(MODE)

where *no* stands for any of the available test problem numbers. This section describes the organization of the FORTRAN subroutines and informs the user how to execute the test problems. Since it is assumed that at least a subset of the problems is used within a series of test runs for different optimization programs, the problems are coded in a very flexible manner. For example, it is possible to compute an arbitrary subset of restriction values. The parameter MODE describes the five possible operations of the subroutine.

MODE=1: The driving program will be provided with all information necessary to initialize an optimization program for the solution of the test problem, i.e. dimension, type and number of constraints, upper and lower bounds, starting point, derivatives of linear constraints, and, in particular, the exact or computed optimal solution.

MODE=2: The objective function $f(x)$ is computed at a current iterate x.

MODE=3: The gradient $\nabla f(x)$ of the objective function will be computed.

MODE=4: A predetermined subset of constraints $g_1(x)$, ..., $g_m(x)$ is evaluated at the actual iterate x.

MODE=5: The gradients of a predetermined subset of nonlinear constraints are computed, i.e. of $\nabla g_1(x)$, ..., $\nabla g_m(x)$.

The information on the test problem is delivered in the following common-blocks which have to be defined in the driving program with appropriate array dimensions:

COMMON/L1/N,NILI,NINL,NELI,NENL: A call of TPno(1) gives on return the data:

N	Dimension of the problem, i.e. n.
NILI	Number of linear inequality constraints, i.e. m_{11}.
NINL	Number of nonlinear inequality constraints, i.e. $m_1 - m_{11}$.
NELI	Number of linear equality constraints, i.e. $m_{21} - m_1$.
NENL	Number of nonlinear equality constraints, i.e. $m - m_{21}$.

COMMON/L2/X(n): For MODE=1, X will be set to a starting point from which the optimization process is to be started. For MODE>1, X must contain the argument x for which the problem functions or derivatives are to be computed.

COMMON/L3/G(m): For all indices J with INDEX1(J)=.TRUE., G(J) is set to the j-th constraint value $g_j(x)$ (MODE=4).

COMMON/L4/GF(n): Contains the partial derivatives of the objective function on return, i.e., GF(I) is set to $\frac{\partial}{\partial x_i} f(x)$, $i = 1, \ldots ,n$ (MODE=3).

COMMON/L5/GG(m,n): For MODE=1, all constant partial derivatives are stored in GG. In particular, the rows 1, …, m_{11} and $m_1 + 1$, …, m_{21} of GG store the constant derivatives of the linear constraints. For MODE=5, the j-th row of GG defined by INDEX2(J)=.TRUE. will be replaced by the gradient of the j-th restriction, i.e. GG(J,I) is set to $\frac{\partial}{\partial x_i} g_j(x)$, if this term is not constant. Since all array dimensions of the common blocks are defined by the exact values of n or m, respectively, we recommend to define GG as a one-dimensional array in the driving program and to use it there in the form GG((I-1)·M+J).

COMMON/L6/FX: For MODE=2, FX contains the objective function value $f(x)$ on return.

COMMON/L9/INDEX1(m): The logical array INDEX1 has to be initialized by the user before calling TPno(4), and defines the restrictions which are to be computed in the case MODE=4. INDEX1 is not changed by the subroutine.

COMMON/L10/INDEX2(m): The logical array INDEX2 has to be initialized by the user before calling TPno(5), and defines the gradients of the nonlinear restrictions which are to be computed in the case MODE=5. INDEX2 is not changed during a call of TPno.

COMMON/L11/LXL(n): The logical array LXL informs about the existence of lower bounds. If there is a lower bound for the i-th variable, LXL(I) is set to .TRUE. during a call of TPno(1). Otherwise, we find LXL(I)=.FALSE..

COMMON/L12/LXU(n): Same for the existence of upper bounds.

COMMON/L13/XL(n): If LXL(I)=.TRUE., XL(I) obtains a lower bound for the i-th variable during a call of TPno(1).

COMMON/L14/XU(n): if LXU(I)=.TRUE., XU(I) is set to an upper bound for the i-th variable during a call of TPno(1).

COMMON/L20/LEX,NEX,FEX,XEX (NEX·n) : L20 contains information on the optimal solution of the problem and is set during a call of TPno(1). If LEX=.FALSE., an exact solution is not known a priori and XEX stores the best computed solution known to the author. Otherwise, we have LEX=.TRUE. and NEX shows the number of all optimal solutions. NEX=-1 indicates that infinitely many solutions are present. FEX contains the minimal objective function value and XEX(J) the J-th optimal solution at positions XEX(N·(J-1)+I), where $i = 1, \ldots, n$ and $j = 1, \ldots,$ NEX. In the case NEX = -1, XEX contains only one arbitrary solution.

Note that in some cases, analytical gradients are not available. There is no warranty that the gradients, as far as included, are correct. Moreover, the test problems have been implemented in a quite elementary form. It might be necessary to set some suitable switches of the Fortran compiler, for example to initialize all variables with zero. In some cases, the implementation differs slightly from the printed documentation in [4] and [9] because of some misprints or some internal modifications to improve numerical stability.

To give an example, we consider Rosenbrock's post office problem, i.e. test problem TP37 of the first collection, [4], given in the form

$$x = (x_1, x_2, x_3)^T \in I\!R^3 : \begin{array}{ll} \min -x_1 x_2 x_3 & \\ x_1 + 2x_2 + 2x_3 - 72 \leq 0, & \\ x_1 + 2x_2 + 2x_3 \geq 0, & \\ 0 \leq x_i \leq 42, & i = 1, 2, 3 \end{array} \tag{2}$$

We have three variables, i.e. $n = 3$, bounds for all variables and only two linear inequality constraints, i.e. $m_{11} = m_1 = m_{21} = m = 3$. The Fortran source code is:

```
      SUBROUTINE TP37(MODE)
      COMMON/L1/N,NILI,NINL,NELI,NENL
      COMMON/L2/X(3)
      COMMON/L3/G(2)
      COMMON/L4/GF(3)
      COMMON/L5/GG(2,3)
      COMMON/L6/FX
      COMMON/L9/INDEX1
      COMMON/L10/INDEX2
      COMMON/L11/LXL
      COMMON/L12/LXU
      COMMON/L13/XL(3)
      COMMON/L14/XU(3)
      COMMON/L20/LEX,NEX,FEX,XEX(3)
      REAL*8 X,G,GF,GG,FX,XL,XU,FEX,XEX
      LOGICAL LEX,LXL(3),LXU(3),INDEX1(2),INDEX2(2)
      GOTO (1,2,3,4,5),MODE
 1    N=3
      NILI=2
      NINL=0
      NELI=0
      NENL=0
      DO 6 I=1,3
```

```
      X(I)=10.D0
      LXL(I)=.TRUE.
      LXU(I)=.TRUE.
      XU(I)=42.D0
 6    XL(I)=0.D0
      LEX=.TRUE.
      NEX=1
      XEX(1)=24.D0
      XEX(2)=12.D0
      XEX(3)=12.D0
      FEX=-3.456D+3
      GG(1,1)=-1.D0
      GG(1,2)=-2.D0
      GG(1,3)=-2.D0
      GG(2,1)=1.D0
      GG(2,2)=2.D0
      GG(2,3)=2.D0
      RETURN
 2    FX=-X(1)*X(2)*X(3)
      RETURN
 3    GF(1)=-X(2)*X(3)
      GF(2)=-X(1)*X(3)
      GF(3)=-X(1)*X(2)
      RETURN
 4    IF (INDEX1(1)) G(1)=72.D0-X(1)-2.D0*X(2)-2.D0*X(3)
      IF (INDEX1(2)) G(2)=X(1)+2.D0*X(2)+2.D0*X(3)
 5    RETURN
      END
```

To show how to call subroutine TP37, we list the corresponding Fortran source code executing NLPQLP.

```
      IMPLICIT   NONE
      INTEGER    NMAX,MMAX,MNNMAX,LWA,LIWA,LACTIV
      PARAMETER (NMAX   = 200,
     F           MMAX   = 200,
     F           MNNMAX = NMAX + NMAX + MMAX + 2,
     F           LWA    = 1.5*NMAX*NMAX + 33*NMAX + 9*MMAX + 200,
     F           LIWA   = NMAX + 10,
     F           LACTIV = 2*MMAX + 10)
      REAL*8     X, G, DF, DG, F, XL, XU, FEX, XEX,
     F           U(MNNMAX), C(NMAX,NMAX), D(NMAX), WA(LWA)
      REAL*8     ACC, ACCQP, TOL_NM, STPMIN
      INTEGER    N, NILI, NINL, NELI, NENL, IWA(LIWA), M, ME, MI,
     F           MNN2, MODE, IPRINT, IOUT, MAXFUN, MAXIT, NEX,
     F           MAX_NM, L, IFAIL, I, J
      LOGICAL    INDEX1, INDEX2, LXL, LXU, LEX, ACTIVE(LACTIV)
      EXTERNAL   QL
      COMMON    /L1/  N, NILI, NINL, NELI, NENL
     F          /L2/  X(NMAX)
     F          /L3/  G(MMAX)
     F          /L4/  DF(NMAX)
     F          /L5/  DG(NMAX*MMAX)
     F          /L6/  F
     F          /L9/  INDEX1(MMAX)
     F          /L10/ INDEX2(MMAX)
     F          /L11/ LXL(NMAX)
     F          /L12/ LXU(NMAX)
     F          /L13/ XL(NMAX)
     F          /L14/ XU(NMAX)
     F          /L20/ LEX, NEX, FEX, XEX(NMAX)
C
C   Optimization settings for NLPQLP
```

```
C
      MODE   = 0
      IPRINT = 2
      IOUT   = 6
      MAXFUN = 20
      MAXIT  = 500
      MAX_NM = 30
      TOL_NM = 0.5D0
      L      = 1
      STPMIN = 1.0D-8
      ACC    = 1.0D-14
      ACCQP  = 1.0D-14
C
C   Model parameters and bounds
C
      CALL TP37(1)
      ME = NELI + NENL
      MI = NILI + NINL
      M  = ME + MI
      DO I=1,N
        IF (.NOT.LXL(I)) XL(I) = X(I) - 1.0D+10
        IF (.NOT.LXU(I)) XU(I) = X(I) + 1.0D+10
        IF (X(I).LT.XL(I)) X(I) = XL(I)
        IF (X(I).GT.XU(I)) X(I) = XU(I)
      ENDDO
      DO J=1,M
        INDEX1(J) = .TRUE.
      ENDDO
C
C   Call of NLPQLP with reverse communication
C
      IFAIL = 0
    1 CONTINUE
      IF (IFAIL.EQ.0.OR.IFAIL.EQ.-1) THEN
         CALL TP37(2)
         CALL TP37(4)
      ENDIF
      IF (IFAIL.EQ.0.OR.IFAIL.EQ.-2) THEN
         CALL TP37(3)
         CALL TP37(5)
      ENDIF
      CALL NLPQLP(L,M,ME,M,N,NMAX,M+N+N+2,X,F,G,DF,DG,U,XL,XU,C,D,
     F            ACC,ACCQP,STPMIN,MAXFUN,MAXIT,MAX_NM,TOL_NM,
     F            IPRINT,MODE,IOUT,IFAIL,WA(M+1),LWA,IWA,LIWA,ACTIVE,
     F            LACTIV,.TRUE.,QL)
      IF (IFAIL.LT.0) GOTO 1
C
C   End
C
      STOP
      END
```

The following output should appear on screen:

```
--------------------------------------------------------------------
 START OF THE SEQUENTIAL QUADRATIC PROGRAMMING ALGORITHM
--------------------------------------------------------------------

 Parameters:
    N      =       3
    M      =       2
    ME     =       0
```

```
    MODE   =       0
    ACC    =   0.1000D-13
    ACCQP  =   0.1000D-13
    STPMIN =   0.1000D-07
    MAXFUN =      20
    MAX_NM =      30
    MAXIT  =     500
    IPRINT =       2

 Output in the following order:
    IT    - iteration number
    F     - objective function value
    SCV   - sum of constraint violations
    NA    - number of active constraints
    I     - number of line search iterations
    ALPHA - steplength parameter
    DELTA - additional variable to prevent inconsistency
    KKT   - Karush-Kuhn-Tucker optimality criterion


  IT         F          SCV      NA  I    ALPHA     DELTA     KKT
--------------------------------------------------------------------
   1 -0.10000000D+04  0.00D+00    2  0  0.00D+00  0.00D+00  0.44D+04
   2 -0.23625000D+04  0.00D+00    1  1  0.10D+01  0.00D+00  0.11D+04
   3 -0.32507304D+04  0.36D-14    1  1  0.10D+01  0.00D+00  0.69D+03
   4 -0.33041403D+04  0.00D+00    1  1  0.10D+01  0.00D+00  0.36D+03
   5 -0.34527380D+04  0.00D+00    1  1  0.10D+01  0.00D+00  0.58D+01
   6 -0.34559629D+04  0.00D+00    1  1  0.10D+01  0.00D+00  0.76D-01
   7 -0.34560000D+04  0.00D+00    1  1  0.10D+01  0.00D+00  0.25D-04
   8 -0.34560000D+04  0.36D-14    1  1  0.10D+01  0.00D+00  0.90D-10
   9 -0.34560000D+04  0.00D+00    1  1  0.10D+01  0.00D+00  0.46D-09
  10 -0.34560000D+04  0.00D+00    1  2  0.10D+00  0.00D+00  0.25D-10
  11 -0.34560000D+04  0.36D-14    1  1  0.10D+01  0.00D+00  0.10D-11
  12 -0.34560000D+04  0.00D+00    1  2  0.50D+00  0.00D+00  0.25D-14

  --- Final Convergence Analysis at Best Iterate ---

    Best result at iteration:      ITER  =      12
    Objective function value:      F(X)  = -0.34560000D+04
    Approximation of solution:     X     =
       0.24000000D+02  0.12000000D+02  0.12000000D+02
    Approximation of multipliers:  U     =
       0.14400000D+03  0.00000000D+00  0.00000000D+00  0.00000000D+00
       0.00000000D+00  0.00000000D+00  0.00000000D+00  0.00000000D+00
    Constraint values:             G(X)  =
       0.00000000D+00  0.72000000D+02
    Distance from lower bound:     XL-X  =
      -0.24000000D+02 -0.12000000D+02 -0.12000000D+02
    Distance from upper bound:     XU-X  =
       0.18000000D+02  0.30000000D+02  0.30000000D+02
    Number of function calls:      NFUNC =      14
    Number of gradient calls:      NGRAD =      12
    Number of calls of QP solver:  NQL   =      12
```

3 Program Organization

All 306 test problems of the two collections [4] and [9] are available together with a test frame. A decision is made which of the runs is successful, and performance results are evaluated. With the default tolerances given, all problems can be solved successfully by the code NLPQLP, a new version of the SQP implementation NLPQL of the author [11].

Results of NLPQLP are discussed in the subsequent section.

The following files are provided by the author and can be downloaded from

`http://www.uni-bayreuth.de/departments/math/~kschittkowski/home.htm`

1. **PROB.FOR**: Fortran codes of the test problems of the two collectoins mentioned above.

2. **CONV.FOR**: Interface between the individual test problem codes and an available optimization routine to facilitate the calling procedure and to be able to execute all test problems within a loop. The subroutine is invoked by

```
CALL CONV(MODE)
```

   where the test problem number is passed through the common block

```
COMMON/L8/NTP
```

3. **TESTP.FOR**: Test program that executes test problems in a loop. The calling sequence for the SQP code NLPQLP is included to give an example. Different approximation formulae for gradient evaluations are included. The code generates the output files listed below.

4. **TEST.DAT**: Output file of the test frame containing numerical results obtained by NLPQLP. Typical contants of TEST.DAT without lines generated by the NLP routine:

TP 1	2	0	0	0	26	19	178	0.00000000E+00	0.73114619E-10	0.73E-10	0.00E+00
TP 2	2	0	0	0	20	15	140	0.50426188E-01	0.50426193E-01	0.11E-06	0.00E+00
TP 3	2	0	0	0	10	10	90	0.00000000E+00	0.16103740E-19	0.16E-19	0.00E+00
TP 4	2	0	0	0	2	2	18	0.26666667E+01	0.26666667E+01	0.00E+00	0.00E+00
TP 5	2	0	0	0	8	6	56	-0.19132230E+01	-0.19132230E+01	0.11E-10	0.00E+00
TP 6	2	1	1	0	10	9	82	0.00000000E+00	0.19130495E-12	0.19E-12	0.22E-04
TP 7	2	1	1	0	11	10	91	-0.17320508E+01	-0.17320508E+01	-0.18E-08	0.11E-07
TP 8	2	2	2	0	5	5	45	-0.10000000E+01	-0.10000000E+01	0.00E+00	0.53E-04

.....

   The following data are listed, see TESTP.FOR for details:

NTP	Test problem number
N	Number of variables
ME	Number of equality constraints
M	Number of constraints
IFAIL	Convergence criterion
NF	Number of objective function evaluations
NDF	Number of gradient evaluations of objective function
NEF	Number of equivalent function evaluations, i.e. NF plus number of function calls needed for gradient approximation
FEX	Exact objective function value
F	Computed objective function value
DFX	Relative error in objective function
DGX	Sum of constraint violations including bound violations

5. **TEST.TEX**: Same as above, but with Latex separators.

6. **RESULT.DAT**: The following summary is shown:

   (a) Flag for evaluating gradients
   (b) Tolerance for gradient approximation
   (c) Termination accuracy for NLP routine
   (d) Randomly generated error added to objective
   (e) Total number of test runs
   (f) Number of successful test runs
   (g) Number of local solutions obtained
   (h) Number of test runs with error messaeg IFAIL>0
   (i) Tolerance for determining successful return
   (j) Average number of function evaluations
   (k) Average number of gradient evaluations
   (l) Average number of equivalent function calls
   (m) Total execution time over all test runs (sec)

7. **TEMP.DAT**: Contains the same data in one row.

4 Numerical Results

The results of some computational tests are reported in this section. They have been obtained by the code NLPQLP [11], a Fortran implementation of a sequential quadratic programming algorithm. The Fortran subroutine NLPQLP solves smooth nonlinear programming problems and is an extension of the code NLPQL, see Schittkowski [8]. The new version is specifically tuned to run under distributed systems and to apply non-monotone line search in error situations. A new input parameter l is introduced for the number of parallel machines, that is the number of function calls to be executed simultaneously. In case of $l = 1$, NLPQLP is more or less identical to NLPQL.

Sequential quadratic programming or SQP methods belong to the most powerful nonlinear programming algorithms we know today for solving differentiable nonlinear programming problems of the form (1). The theoretical background is described for example by Stoer [13] in form of a review, or in Spellucci [12] in form of an extensive text book. From the more practical point of view, SQP methods are also introduced in the books of Papalambros, Wilde [6] and Edgar, Himmelblau [2]. Their excellent numerical performance is evaluated and compared to other methods in Schittkowski [7]. Since many years they belong to the most frequently used algorithms to solve practical optimization problems.

Since analytical derivatives are not available for all problems, we approximate them numerically. The test examples are provided with exact solutions, either known from analytical solutions or from the best numerical data found so far. The Fortran codes are compiled by the Intel Visual Fortran Compiler, Version 9.1, under Windows XP64. Since the calculation times are very short, about one second for solving all test problems, we count only function and gradient evaluations. This is a realistic assumption, since for the practical applications, calculation times for evaluating model functions dominate and the numerical efforts within an optimization code are negligible.

First we need a criterion to decide whether the result of a test run is considered as a successful return or not. Let $\epsilon > 0$ be a tolerance for defining the relative termination accuracy, x_k the final iterate of a test run, and $x^\star$ the supposed exact solution as reported by the two test problem collections. Then we call the output of an execution of NLPQLP a successful return, if the relative error in objective function is less than ϵ and if the sum of all constraint violations less than ϵ^2, i.e., if

$$f(x_k) - f(x^\star) < \epsilon |f(x^\star)| \ , \ \text{if } f(x^\star) \neq 0 \ \ ,$$

or

$$f(x_k) < \epsilon \ , \ \text{if } f(x^\star) = 0 \ \ ,$$

and

$$r(x_k) := \sum_{j=1}^{m_e} |g_j(x_k)| + \sum_{j=m_e+1}^{m} |\min(0, g_j(x_k))| < \epsilon^2 \ \ .$$

We take into account that NLPQLP returns a solution with a better function value than the known one, subject to the error tolerance of the allowed constraint violation. However, there is still the possibility that NLPQLP terminates at a local solution different from the one known in advance. Thus, we call a test run a successful one, if NLPQLP terminates with error message IFAIL=0, and if

$$f(x_k) - f(x^\star) \geq \epsilon |f(x^\star)| \ , \ \text{if } f(x^\star) \neq 0 \ \ ,$$

or

$$f(x_k) \geq \epsilon \ , \ \text{if } f(x^\star) = 0 \ \ ,$$

and

$$r(x_k) < \epsilon^2 \ \ .$$

For our numerical tests, we use $\epsilon = 0.01$, i.e., we require a final accuracy of one per cent. NLPQLP is executed with termination accuracy *ACC*=10^{-7}, and *MAXIT=500.* Gradients are approximated by forward differences. Neither variables nor functions are scaled internally. All problems are executed with one and the same set of solution tolerances.

When executing NLPQLP for the 115 test examples of the first collection of Hock and Schittkowski [4], the following results are obtained:

Flag for evaluating gradients	:	1
Tolerance for gradient approximation	:	0.1D-07
Termination accuracy for NLP routine	:	0.1D-06
Randomly generated error added to objective	:	0.0D+00
Total number of test runs	:	115
Number of successful test runs	:	115
Number of better test runs	:	0
Number of local solutions obtained	:	7
Number of runs without satisfying termination accuracy	:	0
Tolerance for determining successful return	:	0.1D-01
Average number of function evaluations	:	26
Average number of gradient evaluations	:	16
Average number of equivalent function calls	:	122
Total execution time over all test runs	:	0.12 (sec)

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APPENDIX: Individual Results

The appendix contains a list of all test problems with the data

TP	test problem number,
N	number of variables,
ME	number of equality constraints,
M	number of constraints,
IFAIL	convergence criterion,
NF	number of objective function evaluations,
NDF	number of gradient evaluations of objective function,
NEF	number of equivalent function evaluations, i.e. NF plus number of function calls needed for gradient approximation,
FEX	exact objective function value,
F	computed objective function value,
DFX	relative error in objective function,
DGX	sum of constraint violations including bound violations.

The performance results are obtained by NLPQLP under the conditions outlined in Section 4.

TP	*N*	*ME*	*M*	*IFAIL*	*NF*	*NDF*	*NEF*	*FEX*	*F*	*DFX*	*DGX*
1	2	0	0	0	26	19	64	0.00000000E+00	0.58256090E-10	0.58E-10	0.00E+00
2	2	0	0	0	20	15	50	0.50426188E-01	0.50426193E-01	0.10E-06	0.00E+00
3	2	0	0	0	10	10	30	0.00000000E+00	0.23971330E-17	0.24E-17	0.00E+00
4	2	0	0	0	2	2	6	0.26666667E+01	0.26666667E+01	0.00E+00	0.00E+00
5	2	0	0	0	8	6	20	-0.19132230E+01	-0.19132230E+01	0.11E-10	0.00E+00

(continued)

TP	*N*	*ME*	*M*	*IFAIL*	*NF*	*NDF*	*NEF*	*FEX*	*F*	*DFX*	*DGX*
6	2	1	1	0	10	9	28	0.00000000E+00	0.18671535E-12	0.19E-12	0.22E-04
7	2	1	1	0	11	10	31	-0.17320508E+01	-0.17320508E+01	-0.86E-09	0.51E-08
8	2	2	2	0	5	5	15	-0.10000000E+01	-0.10000000E+01	0.00E+00	0.53E-04
9	2	1	1	0	6	6	18	-0.50000000E+00	-0.50000000E+00	0.55E-09	0.18E-14
10	2	0	1	0	13	12	37	-0.10000000E+01	-0.10000000E+01	-0.45E-10	0.91E-10
11	2	0	1	0	10	9	28	-0.84984642E+01	-0.84984642E+01	-0.30E-12	0.84E-12
12	2	0	1	0	9	8	25	-0.30000000E+02	-0.30000000E+02	-0.58E-09	0.35E-07
13	2	0	1	0	42	42	126	0.10000000E+01	0.10000001E+01	0.98E-07	0.00E+00
14	2	1	2	0	6	6	18	0.13934650E+01	0.13934650E+01	-0.10E-11	0.76E-12
15	2	0	2	0	3	3	9	0.30650001E+01	0.30650000E+01	-0.23E-07	0.19E-08
16	2	0	2	0	12	8	28	0.25000000E+00	0.39820605E+01	0.15E+02	0.00E+00
17	2	0	2	0	20	17	54	0.10000000E+01	0.10000000E+01	0.28E-11	0.00E+00
18	2	0	2	0	8	8	24	0.50000000E+01	0.50000000E+01	-0.11E-08	0.39E-07
19	2	0	2	0	7	7	21	-0.69618139E+04	-0.69618139E+04	-0.22E-14	0.36E-14
20	2	0	3	0	5	5	15	0.38198730E+02	0.38198730E+02	0.18E-09	0.00E+00
21	2	0	1	0	3	2	7	-0.99960000E+02	-0.99960000E+02	-0.89E-11	0.00E+00
22	2	0	2	0	7	6	19	0.10000000E+01	0.10000000E+01	-0.22E-11	0.33E-11
23	2	0	5	0	7	7	21	0.20000000E+01	0.20000000E+01	0.30E-13	0.00E+00
24	2	0	3	0	5	5	15	-0.10000000E+01	-0.10000000E+01	-0.17E-07	0.41E-07
25	3	0	0	0	1	1	4	0.00000000E+00	0.32835000E+02	0.33E+02	0.00E+00
26	3	1	1	0	20	18	74	0.00000000E+00	0.61599042E-07	0.62E-07	0.59E-04
27	3	1	1	0	37	22	103	0.40000000E+01	0.40000000E+01	0.42E-10	0.25E-12
28	3	1	1	0	5	4	17	0.00000000E+00	0.35555652E-13	0.36E-13	0.44E-07
29	3	0	1	0	13	12	49	-0.22627417E+02	-0.22627417E+02	-0.21E-09	0.69E-08
30	3	0	1	0	18	16	66	0.10000000E+01	0.10000000E+01	0.76E-09	0.00E+00
31	3	0	1	0	12	7	33	0.60000000E+01	0.60000000E+01	-0.51E-08	0.80E-08
32	3	1	2	0	3	3	12	0.10000000E+01	0.10000000E+01	0.65E-08	0.33E-08
33	3	0	2	0	5	5	20	-0.45857864E+01	-0.40000000E+01	0.13E+00	0.00E+00
34	3	0	2	0	8	8	32	-0.83403245E+00	-0.83403245E+00	-0.12E-09	0.11E-08
35	3	0	1	0	7	7	28	0.11111111E+00	0.11111111E+00	0.54E-11	0.00E+00
36	3	0	1	0	10	4	22	-0.33000000E+04	-0.33000000E+04	0.90E-14	0.00E+00
37	3	0	2	0	12	10	42	-0.34560000E+04	-0.34560000E+04	-0.15E-12	0.36E-11
38	4	0	0	0	111	84	447	0.00000000E+00	0.16953149E-08	0.17E-08	0.00E+00
39	4	2	2	0	14	12	62	-0.10000000E+01	-0.10000000E+01	-0.41E-08	0.25E-08
40	4	3	3	0	6	6	30	-0.25000000E+00	-0.25000000E+00	-0.93E-09	0.43E-09
41	4	1	1	0	8	8	40	0.19259259E+01	0.19259259E+01	0.75E-10	0.10E-10
42	4	2	2	0	10	8	42	0.13857864E+02	0.13857864E+02	-0.31E-08	0.17E-07
43	4	0	3	0	14	11	58	-0.44000000E+02	-0.44000000E+02	-0.35E-10	0.64E-09
44	4	0	6	0	6	6	30	-0.15000000E+02	-0.15000000E+02	-0.27E-08	0.27E-07
45	5	0	0	0	8	8	48	0.10000000E+01	0.10000000E+01	0.00E+00	0.00E+00
46	5	2	2	0	14	12	74	0.00000000E+00	0.55478753E-06	0.55E-06	0.19E-06
47	5	3	3	0	17	13	82	0.00000000E+00	0.46084429E-09	0.46E-09	0.93E-07
48	5	2	2	0	9	8	49	0.00000000E+00	0.27709272E-07	0.28E-07	0.80E-10
49	5	2	2	0	9	6	39	0.00000000E+00	0.22835655E-04	0.23E-04	0.15E-09
50	5	3	3	0	18	14	88	0.00000000E+00	0.31210594E-08	0.31E-08	0.11E-11
51	5	3	3	0	5	3	20	0.00000000E+00	0.23430051E-15	0.23E-15	0.19E-07
52	5	3	3	0	8	6	38	0.53266476E+01	0.53266476E+01	0.40E-10	0.55E-12
53	5	3	3	0	8	7	43	0.40930233E+01	0.40930233E+01	0.74E-08	0.51E-10
54	6	1	1	0	2	2	14	-0.90807476E+00	-0.72239851E-33	0.10E+01	0.14E-05
55	6	6	6	0	2	2	14	0.63333333E+01	0.66666667E+01	0.53E-01	0.15E-06
56	7	4	4	0	11	9	74	-0.34560000E+01	-0.34560000E+01	-0.10E-07	0.24E-07
57	2	0	1	0	5	3	11	0.28459670E-01	0.30646306E-01	0.77E-01	0.00E+00
59	2	0	3	0	17	15	47	-0.78042263E+01	-0.67545660E+01	0.13E+00	0.00E+00
60	3	1	1	0	11	10	41	0.32568200E-01	0.32568200E-01	0.23E-08	0.67E-08
61	3	2	2	0	8	7	29	-0.14364614E+03	-0.14364614E+03	-0.11E-09	0.18E-07
62	3	1	1	0	14	10	44	-0.26272514E+05	-0.26272514E+05	-0.69E-12	0.18E-13
63	3	2	2	0	10	9	37	0.96171517E+03	0.96171517E+03	0.32E-11	0.78E-11

(continued)

TP	*N*	*ME*	*M*	*IFAIL*	*NF*	*NDF*	*NEF*	*FEX*	*F*	*DFX*	*DGX*
64	3	0	1	0	49	33	148	0.62998424E+04	0.62998424E+04	-0.52E-10	0.17E-10
65	3	0	1	0	8	8	32	0.95352886E+00	0.95352882E+00	-0.42E-07	0.54E-06
66	3	0	2	0	7	7	28	0.51816327E+00	0.51816327E+00	-0.22E-08	0.31E-08
67	3	0	14	0	20	20	80	-0.11620365E+04	-0.11620365E+04	-0.15E-07	0.00E+00
68	4	2	2	0	40	26	144	-0.92042502E+00	-0.92042504E+00	-0.18E-07	0.11E-06
69	4	2	2	0	50	32	178	-0.95671289E+03	-0.95671289E+03	0.43E-09	0.11E-10
70	4	0	1	0	37	34	173	0.74984636E-02	0.74984827E-02	0.26E-05	0.00E+00
71	4	1	2	0	5	5	25	0.17014017E+02	0.17014017E+02	-0.26E-08	0.22E-08
72	4	0	2	0	22	22	110	0.72767938E+03	0.72767936E+03	-0.25E-07	0.11E-11
73	4	1	3	0	5	5	25	0.29894378E+02	0.29894378E+02	0.62E-10	0.35E-11
74	4	3	5	0	10	10	50	0.51264981E+04	0.51264981E+04	0.50E-10	0.15E-09
75	4	3	5	0	9	9	45	0.51744129E+04	0.51744127E+04	-0.37E-07	0.27E-11
76	4	0	3	0	6	6	30	-0.46818182E+01	-0.46818182E+01	0.49E-10	0.00E+00
77	5	2	2	0	16	15	91	0.24150513E+00	0.24150513E+00	-0.14E-07	0.35E-07
78	5	3	3	0	8	8	48	-0.29197004E+01	-0.29197004E+01	0.50E-10	0.29E-11
79	5	3	3	0	10	9	55	0.78776821E-01	0.78776822E-01	0.10E-07	0.30E-07
80	5	3	3	0	7	7	42	0.53949848E-01	0.53949847E-01	-0.73E-08	0.72E-08
81	5	3	3	0	8	8	48	0.53949848E-01	0.53949846E-01	-0.28E-07	0.27E-07
83	5	0	6	0	5	5	30	-0.30665539E+05	-0.30665539E+05	-0.27E-11	0.00E+00
84	5	0	6	0	10	10	60	-0.52803351E+02	-0.52803351E+02	-0.29E-10	0.93E-14
85	5	0	38	0	91	56	371	-0.19051338E+01	-0.19051553E+01	-0.11E-04	0.15E-06
86	5	0	10	0	6	5	31	-0.32348679E+02	-0.32348679E+02	0.20E-09	0.11E-15
87	6	4	4	0	20	16	116	0.89275977E+04	0.89275977E+04	0.60E-10	0.54E-08
88	2	0	1	0	24	18	60	0.13626568E+01	0.13626907E+01	0.25E-04	0.26E-12
89	3	0	1	0	42	27	123	0.13626568E+01	0.13626907E+01	0.25E-04	0.45E-11
90	4	0	1	0	59	26	163	0.13626568E+01	0.13626907E+01	0.25E-04	0.20E-11
91	5	0	1	0	47	33	212	0.13626568E+01	0.13626909E+01	0.25E-04	0.13E-10
92	6	0	1	0	50	36	266	0.13626568E+01	0.13626907E+01	0.25E-04	0.00E+00
93	6	0	2	0	15	12	87	0.13507596E+03	0.13507596E+03	0.12E-07	0.41E-09
95	6	0	4	0	2	2	14	0.15619514E-01	0.15619525E-01	0.69E-06	0.18E-08
96	6	0	4	0	2	2	14	0.15619513E-01	0.15619525E-01	0.76E-06	0.18E-08
97	6	0	4	0	7	7	49	0.31358091E+01	0.31358091E+01	-0.29E-08	0.35E-07
98	6	0	4	0	7	7	49	0.31358091E+01	0.31358091E+01	-0.29E-08	0.35E-07
99	7	2	2	0	279	46	601	-0.83107989E+09	-0.83107989E+09	0.71E-11	0.17E-09
100	7	0	4	0	20	14	118	0.68063006E+03	0.68063006E+03	0.11E-09	0.25E-07
101	7	0	6	0	70	42	364	0.18097648E+04	0.18097648E+04	0.51E-11	0.17E-11
102	7	0	6	0	52	36	304	0.91188057E+03	0.91188057E+03	0.10E-09	0.39E-12
103	7	0	6	0	46	31	263	0.54366796E+03	0.54366796E+03	0.75E-10	0.86E-12
104	8	0	6	0	16	16	144	0.39511634E+01	0.39511634E+01	0.44E-09	0.57E-08
105	8	0	1	0	53	46	421	0.11384162E+04	0.11384185E+04	0.20E-05	0.00E+00
106	8	0	6	0	609	226	2417	0.70493309E+04	0.70492480E+04	-0.12E-04	0.16E-06
107	9	6	6	0	8	8	80	0.50550118E+04	0.50550114E+04	-0.70E-07	0.74E-13
108	9	0	13	0	13	13	130	-0.86602540E+00	-0.86602540E+00	-0.81E-09	0.16E-08
109	9	6	10	0	56	19	227	0.53620693E+04	0.53620692E+04	-0.18E-07	0.57E-12
110	10	0	0	0	12	8	92	-0.45778470E+02	-0.45778470E+02	0.41E-09	0.00E+00
111	10	3	3	0	51	51	561	-0.47761090E+02	-0.47761091E+02	-0.13E-07	0.12E-09
112	10	3	3	0	39	21	249	-0.47761086E+02	-0.47761091E+02	-0.10E-06	0.33E-11
113	10	0	8	0	16	13	146	0.24306209E+02	0.24306209E+02	0.16E-09	0.24E-08
114	10	3	11	0	42	33	372	-0.17688070E+04	-0.17688070E+04	-0.16E-09	0.23E-12
116	13	0	15	0	100	68	984	0.97588409E+02	0.97587510E+02	-0.92E-05	0.17E-08
117	15	0	5	0	16	16	256	0.32348679E+02	0.32348679E+02	0.49E-09	0.00E+00
118	15	0	29	0	20	20	320	0.66482045E+03	0.66482045E+03	0.11E-10	0.00E+00
119	16	8	8	0	30	16	286	0.24489970E+03	0.24489970E+03	-0.13E-10	0.57E-09

APPENDIX: Test Problems of the First Collection

Purpose of this appendix is to list a detailed description of all test problems published in the monograph [4], which is out of print. We proceed from the nonlinear program (1) and list the following data of an example:

PROBLEM:	test problem number
CLASSIFICATION:	classification number in the form OCD-Kr-s according to the scheme given below
NUMBER OF VARIABLES:	number of variables n
NUMBER OF CONSTRAINTS:	number of inequality constraints, m_1, number of equality constraints, i.e., $m - m_1$, and number of variable bounds of variables, b
OBJECTIVE FUNCTION:	analytical expressions for objective function $f(x)$
CONSTRAINTS:	analytical expressions for constraints $g_j(x), j = 1, \ldots, m$
START:	starting values for variables, x_0, and corresponding objective function value, $f(x_0)$, together with an information whether x_0 is feasible or not
SOLUTION:	information about optimal solution $x^\star$, i.e., - objective function value $f(x^\star)$ - constraint violation, $r(x^\star)$ - norm of gradient of Lagrange function - number of active constraints, μ - active constraints, $I(x^\star)$ - degree of degeneracy, $u^\star_{max}/u^\star_{min}$ - condition number of projected Hessian of Lagrange function, $\lambda^\star_{max}/\lambda^\star_{min}$

The general form of the classification scheme is

OCD-Kr-s

with

O	-	objective function
C	-	constraints
D	-	regularity
K	-	information about solution, i.e., whether an exact solution is known or not
r	-	order of partial derivatives
s	-	serial number within a class

The purpose of the classification scheme is to characterize the mathematical structure of objective function and constraints, and to give more information about the implementation and the solution. A problem is called a regular one, if first and second derivatives exist in the feasible region for all problem functions, otherwise an irregular one. The subsequent abbreviations are used:

class	key	description
O	C	constant function
	L	linear function
	Q	quadratic function
	S	sum of squares
	P	generalized polynomial function
	G	general function
C	U	unconstrained problem
	B	only upper and lower bounds
	L	linear functions
	Q	quadratic functions
	P	generalized polynomial functions
	G	general functions
D	R	regular problem
	I	irregular problem
K	T	exact solution known (*theoretical problem*)
	P	exact solution not known (*practical problem*)
r	0	derivatives not implemented
	1	first derivatives implemented

For some test problems, we cannot describe objective or constraint functions just by a few analytical expressions. In these cases, program fragments are attached at the end of this section together with more extensive information about a test problem, e.g., constant data, starting or solution values.

The subsequent pages are xeroxed copies of the original publication.

PROBLEM:	1
CLASSIFICATION:	PBR-T1-1
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 1$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS:

$$-1.5 \le x_2$$

START:	x_o	=	$(-2\ ,\ 1)$	(feasible)
	$f(x_o)$	=	909	
SOLUTION:	x^*	=	$(1\ ,\ 1)$	
	$f(x^*)$	=	0	
	$r(x^*)$	=	0	
	$e(x^*)$	=	-	
	μ	=	0	
	$I(x^*)$	=	-	
	u^*_{max}/u^*_{min}	=	-	
	$\lambda^*_{max}/\lambda^*_{min}$	=	.25E 4	

PROBLEM:	2
CLASSIFICATION:	PBR-T1-2
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 1$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS:

$$1.5 \le x_2$$

START: $x_o = (-2 , 1)$ (not feasible)

$f(x_o) = 909$

SOLUTION: $x^* = (2a \cos(\frac{1}{3} \arccos\frac{1}{b}) , 1.5)$

$f(x^*) = .05042\ 61879$

$r(x^*) = 0$

$e(x^*) = .13E-7$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .1833/.1833 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 200/200 = 1$

$a = (598/1200)^{1/2}$

$b = 400\ a^3$

PROBLEM:	3
CLASSIFICATION:	QBR-T1-1
SOURCE:	Schuldt [56]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 1$

OBJECTIVE FUNCTION:

$$f(x) = x_2 + 10^{-5}(x_2 - x_1)^2$$

CONSTRAINTS:

$$0 \le x_2$$

START:	x_o	=	$(10 , 1)$	(feasible)
	$f(x_o)$	=	1.00081	
SOLUTION:	x^*	=	$(0 , 0)$	
	$f(x^*)$	=	0	
	$r(x^*)$	=	0	
	$e(x^*)$	=	0	
	μ	=	1	
	$I(x^*)$	=	(1)	
	u^*_{max}/u^*_{min}	=	1.0000/1.0000 = 1	
	$\lambda^*_{max}/\lambda^*_{min}$	=	.20E-4/.20E-4 = 1	

PROBLEM:	4		
CLASSIFICATION:	PBR-T1-3		
SOURCE:	Asaadi [1]		
NUMBER OF VARIABLES:	$n = 2$		
NUMBER OF CONSTRAINTS:	$m_1 = 0$	, $m-m_1 = 0$	, $b = 2$

OBJECTIVE FUNCTION:

$$f(x) = \frac{1}{3}(x_1 + 1)^3 + x_2$$

CONSTRAINTS:

$$1 \le x_1$$

$$0 \le x_2$$

START:	x_o	=	$(1.125\ ,\ .125)$	(feasible)
	$f(x_o)$	=	3.323568	
SOLUTION:	x^*	=	$(1\ ,\ 0)$	
	$f(x^*)$	=	$8/3$	
	$r(x^*)$	=	0	
	$e(x^*)$	=	0	
	μ	=	2	
	$I(x^*)$	=	$(1\ ,\ 2)$	
	u^*_{max}/u^*_{min}	=	$4.0000/1.0000 = 4.00$	
	$\lambda^*_{max}/\lambda^*_{min}$	=	$-$	

PROBLEM:	5
CLASSIFICATION:	GBR-T1-1
SOURCE:	McCormick [41]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m - m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = \sin(x_1 + x_2) + (x_1 - x_2)^2 - 1.5x_1 + 2.5x_2 + 1$$

CONSTRAINTS:

$$-1.5 \leq x_1 \leq 4$$

$$-3 \leq x_2 \leq 3$$

START:	x_o	=	$(0 , 0)$	(feasible)
	$f(x_o)$	=	1	
SOLUTION:	x^*	=	$\left(-\frac{\pi}{3} + \frac{1}{2} , -\frac{\pi}{3} - \frac{1}{2}\right)$	
	$f(x^*)$	=	$-\frac{1}{2}\sqrt{3} - \frac{\pi}{3}$	
	$r(x^*)$	=	0	
	$e(x^*)$	=	-	
	μ	=	0	
	$I(x^*)$	=	-	
	u^*_{max}/u^*_{min}	=	-	
	$\lambda^*_{max}/\lambda^*_{min}$	=	$4.00/1.73 = 2.31$	

PROBLEM:	6
CLASSIFICATION:	QQR-T1-1
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (1 - x_1)^2$$

CONSTRAINTS:

$$10(x_2 - x_1^2) = 0$$

START:	x_o	=	$(-1.2 , 1)$	(not feasible)
	$f(x_o)$	=	4.84	
SOLUTION:	x^*	=	$(1 , 1)$	
	$f(x^*)$	=	0	
	$r(x^*)$	=	0	
	$e(x^*)$	=	0	
	μ	=	0	
	$I(x^*)$	=	-	
	u^*_{max}/u^*_{min}	=	0	
	$\lambda^*_{max}/\lambda^*_{min}$	=	$.40/.40 = 1$	

PROBLEM:	7		
CLASSIFICATION:	GPR-T1-1		
SOURCE:	Miele e.al. [44,45]		
NUMBER OF VARIABLES:	$n = 2$		
NUMBER OF CONSTRAINTS:	$m_1 = 0$	, $m-m_1 = 1$	, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = \ln(1 + x_1^2) - x_2$$

CONSTRAINTS:

$$(1 + x_1^2)^2 + x_2^2 - 4 = 0$$

START:	x_o	=	$(2\ ,\ 2)$	(not feasible)
	$f(x_o)$	=	$\ln 5 - 2$	
SOLUTION:	x^*	=	$(0\ ,\ \sqrt{3})$	
	$f(x^*)$	=	$-\sqrt{3}$	
	$r(x^*)$	=	0	
	$e(x^*)$	=	.21E-24	
	μ	=	0	
	$I(x^*)$	=	–	
	u^*_{max}/u^*_{min}	=	$.2887/.2887 = 1$	
	$\lambda^*_{max}/\lambda^*_{min}$	=	$3.15/3.15 = 1$	

PROBLEM:	8
CLASSIFICATION:	CQR-T1-1
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = -1$$

CONSTRAINTS:

$$x_1^2 + x_2^2 - 25 = 0$$

$$x_1 x_2 - 9 = 0$$

START:

$x_o = (2\ ,\ 1)$ (not feasible)

$f(x_o) = -1$

SOLUTION:

$x^* = (a\ ,\ \frac{9}{a})\ ,\ (-a\ ,-\frac{9}{a})\ ,\ (b\ ,\ \frac{9}{b})\ ,\ (-b\ ,-\frac{9}{b})$

$f(x^*) = -1$

$r(x^*) = 0$ $\qquad a = \sqrt{\frac{25 + \sqrt{301}}{2}}$

$e(x^*) = 0$

$\mu = 0$ $\qquad b = \sqrt{\frac{25 - \sqrt{301}}{2}}$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	9
CLASSIFICATION:	GLR-T1-1
SOURCE:	Miele e.al. [44]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1(1)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = \sin(\pi x_1/12)\cos(\pi x_2/16)$$

CONSTRAINTS:

$$4x_1 - 3x_2 = 0$$

START: $x_o = (0\ ,\ 0)$ (feasible)

$f(x_o) = 0$

SOLUTION: $x^* = (12k - 3\ ,\ 16k - 4)$, $k=0, \pm 1, \pm 2, \ldots$

$f(x^*) = -.5$

$r(x^*) = 0$

$e(x^*) = .73E-12$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .03272/.03272 = 1$

$\lambda^*_{max}/\lambda^*_{min} = .049/.049 = 1$

PROBLEM:	10
CLASSIFICATION:	LQR-T1-1
SOURCE:	Biggs [10]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = x_1 - x_2$$

CONSTRAINTS:

$$-3x_1^2 + 2x_1x_2 - x_2^2 + 1 \geq 0$$

START:

$x_o = (-10 , 10)$ (not feasible)

$f(x_o) = -20$

SOLUTION:

$x^* = (0 , 1)$

$f(x^*) = -1$

$r(x^*) = 0$

$e(x^*) = .92E-11$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .5000/.5000 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 1.00/1.00 = 1$

PROBLEM:	11
CLASSIFICATION:	QQR-T1-2
SOURCE:	Biggs [10]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 5)^2 + x_2^2 - 25$$

CONSTRAINTS:

$$-x_1^2 + x_2 \geq 0$$

START: $x_0 = (4.9\ ,\ .1)$. (not feasible)

$f(x_0) = -24.98$

SOLUTION: $x^* = ((a - \frac{1}{a})/\sqrt{6}\ ,\ (a^2 - 2 + a^{-2})/6)$

$f(x^*) = -8.49846\ 4223$

$r(x^*) = 0$ $\quad a = 7.5\sqrt{6} + \sqrt{338.5}$

$e(x^*) = .17E-9$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = 3.0493/3.0493 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 2.86/2.86 = 1$

PROBLEM:	12
CLASSIFICATION:	QQR-T1-3
SOURCE:	Mine e.al. [46]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = .5x_1^2 + x_2^2 - x_1x_2 - 7x_1 - 7x_2$$

CONSTRAINTS:

$$25 - 4x_1^2 - x_2^2 \geq 0$$

START: $x_o = (0 , 0)$ (feasible)

$f(x_o) = 0$

SOLUTION: $x^* = (2 , 3)$

$f(x^*) = -30$

$r(x^*) = 0$

$e(x^*) = .81E-10$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .5000/.5000 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 3.90/3.90 = 1$

PROBLEM:	13
CLASSIFICATION:	QPR-T1-1
SOURCE:	Betts [8], Kuhn, Tucker [38]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 2$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 2)^2 + x_2^2$$

CONSTRAINTS:

$$(1 - x_1)^3 - x_2 \geq 0$$

$$0 \leq x_1$$

$$0 \leq x_2$$

START:	x_o	=	$(-2 , -2)$ (not feasible)
	$f(x_o)$	=	20
SOLUTION:	x^*	=	$(1 , 0)$
	$f(x^*)$	=	1
	$r(x^*)$	=	0
	$e(x^*)$	=	2 (constraint qualification not satisfied)
	μ	=	2
	$I(x^*)$	=	$(1,3)$
	u^*_{max}/u^*_{min}	=	0/0
	$\lambda^*_{max}/\lambda^*_{min}$	=	-

PROBLEM:	14
CLASSIFICATION:	QQR-T1-4
SOURCE:	Bracken, McCormick [13], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 1(1)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 2)^2 + (x_2 - 1)^2$$

CONSTRAINTS:

$$-.25x_1^2 - x_2^2 + 1 \geq 0$$

$$x_1 - 2x_2 + 1 = 0$$

START: $x_o = (2 , 2)$ (not feasible)

$f(x_o) = 1$

SOLUTION: $x^* = (.5(\sqrt{7} - 1) , .25(\sqrt{7} + 1))$

$f(x^*) = 9 - 2.875\sqrt{7}$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = 1.8466/1.5945 = 1.15$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	15
CLASSIFICATION:	PQR-T1-1
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 1$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS:

$$x_1 x_2 - 1 \geq 0$$

$$x_1 + x_2^2 \geq 0$$

$$x_1 \leq .5$$

START:

$x_0 = (-2 , 1)$ (not feasible)

$f(x_0) = 909$

SOLUTION:

$x^* = (.5 , 2)$

$f(x^*) = 306.5$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 2$

$I(x^*) = (1 , 3)$

$u^*_{max}/u^*_{min} = 1751/700 = 2.50$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	16
CLASSIFICATION:	PQR-T1-2
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 3$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS:

$$x_1 + x_2^2 \geq 0$$

$$x_1^2 + x_2 \geq 0$$

$$-2 \quad -\cancel{.5} \leq x_1 \leq .5$$

$$x_2 \leq 1$$

START: $x_0 = (-2 , 1)$ (not feasible)

$f(x_0) = 909$

SOLUTION: $x^* = (.5 , .25)$

$f(x^*) = .25$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 1$

$I(x^*) = (4)$

$u^*_{max}/u^*_{min} = 1.0000/1.0000 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 200/200 = 1$

PROBLEM:	17
CLASSIFICATION:	PQR-T1-3
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 3$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS:

$$x_2^2 - x_1 \geq 0$$

$$x_1^2 - x_2 \geq 0$$

$$-.5 \leq x_1 \leq .5$$

$$x_2 \leq 1$$

START:	x_o	=	(-2 , 1) (not feasible)
	$f(x_o)$	=	909
SOLUTION:	x^*	=	(0 , 0)
	$f(x^*)$	=	1
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	2
	$I(x^*)$	=	(1 , 2)
	u^*_{max}/u^*_{min}	=	2.0000/0
	$\lambda^*_{max}/\lambda^*_{min}$	=	-

PROBLEM:	18
CLASSIFICATION:	QQR-T1-5
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = .01x_1^2 + x_2^2$$

CONSTRAINTS:

$$x_1x_2 - 25 \geq 0$$

$$x_1^2 + x_2^2 - 25 \geq 0$$

$$2 \leq x_1 \leq 50$$

$$0 \leq x_2 \leq 50$$

START:	x_o	=	(2 , 2) (not feasible)
	$f(x_o)$	=	4.04
SOLUTION:	x^*	=	$(\sqrt{250}\ ,\ \sqrt{2.5})$
	$f(x^*)$	=	5
	$r(x^*)$	=	0
	$e(x^*)$	=	.24E-9
	μ	=	1
	$I(x^*)$	=	(1)
	u^*_{max}/u^*_{min}	=	.2000/.2000 = 1
	$\lambda^*_{max}/\lambda^*_{min}$	=	.079/.079 = 1

PROBLEM:	19
CLASSIFICATION:	PQR-T1-4
SOURCE:	Betts [8], Gould [27]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 10)^3 + (x_2 - 20)^3$$

CONSTRAINTS:

$$(x_1 - 5)^2 + (x_2 - 5)^2 - 100 \geq 0$$

$$-(x_2 - 5)^2 - (x_1 - 6)^2 + 82.81 \geq 0$$

$$13 \leq x_1 \leq 100$$

$$0 \leq x_2 \leq 100$$

START:	x_o	=	(20.1 , 5.84) (not feasible)
	$f(x_o)$	=	-1808.858296
SOLUTION:	x^*	=	(14.095 , .84296079)
	$f(x^*)$	=	-6961.81381
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	2
	$I(x^*)$	=	(1 , 2)
	u^*_{max}/u^*_{min}	=	1229.5/1097.1 = 1.12
	$\lambda^*_{max}/\lambda^*_{min}$	=	-

PROBLEM:	20
CLASSIFICATION:	PQR-T1-5
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 3$, $m-m_1 = 0$, $b = 2$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

CONSTRAINTS:

$$x_1 + x_2^2 \geq 0$$

$$x_1^2 + x_2 \geq 0$$

$$x_1^2 + x_2^2 - 1 \geq 0$$

$$-.5 \leq x_1 \leq .5$$

START:	x_0	=	$(-2 , 1)$ (not feasible)
	$f(x_0)$	=	909
SOLUTION:	x^*	=	$(.5 , .5\sqrt{3})$
	$f(x^*)$	=	$81.5 - 25\sqrt{3}$
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	2
	$I(x^*)$	=	$(3 , 5)$
	u^*_{max}/u^*_{min}	=	$195.34/71.132 = 2.75$
	$\lambda^*_{max}/\lambda^*_{min}$	=	-

PROBLEM:	21
CLASSIFICATION:	QLR-T1-1
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1(1)$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = .01x_1^2 + x_2^2 - 100$$

CONSTRAINTS:

$$10x_1 - x_2 - 10 \geq 0$$

$$2 \leq x_1 \leq 50$$

$$-50 \leq x_2 \leq 50$$

START:

$x_o = (-1 , -1)$ (not feasible)

$f(x_o) = -98.99$

SOLUTION:

$x^* = (2 , 0)$

$f(x^*) = -99.96$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 1$

$I(x^*) = (2)$

$u^*_{max}/u^*_{min} = .04/.04 = 1$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	22
CLASSIFICATION:	QQR-T1-6
SOURCE:	Bracken, McCormick [13], Himmelblau [29], Sheela [57]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 2(1)$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 2)^2 + (x_2 - 1)^2$$

CONSTRAINTS:

$$-x_1 - x_2 + 2 \geq 0$$

$$-x_1^2 + x_2 \geq 0$$

START: $x_o = (2 , 2)$ (not feasible)

$f(x_o) = 1$

SOLUTION: $x^* = (1 , 1)$

$f(x^*) = 1$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 2$

$I(x^*) = (1 , 2)$

$u^*_{max}/u^*_{min} = .6666/.6666 = 1$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	23
CLASSIFICATION:	QQR-T1-7
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 5(1)$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = x_1^2 + x_2^2$$

CONSTRAINTS:

$$x_1 + x_2 - 1 \geq 0$$

$$x_1^2 + x_2^2 - 1 \geq 0$$

$$9x_1^2 + x_2^2 - 9 \geq 0$$

$$x_1^2 - x_2 \geq 0$$

$$x_2^2 - x_1 \geq 0$$

$$-50 \leq x_i \leq 50 \quad , \quad i=1,2$$

START: $x_o = (3 , 1)$ (not feasible)

$f(x_o) = 10$

SOLUTION:

$x^* = (1 , 1)$

$f(x^*) = 2$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 2$

$I(x^*) = (4 , 5)$

$u^*_{max}/u^*_{min} = 2/2 = 1$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	24
CLASSIFICATION:	PLR-T1-1
SOURCE:	Betts [8], Box [12]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 3(3)$, $m-m_1 = 0$, $b = 2$

OBJECTIVE FUNCTION:

$$f(x) = \frac{1}{27\sqrt{3}} ((x_1 - 3)^2 - 9)\, x_2^3$$

CONSTRAINTS:

$$x_1/\sqrt{3} - x_2 \geq 0$$

$$x_1 + \sqrt{3}x_2 \geq 0$$

$$-x_1 - \sqrt{3}x_2 + 6 \geq 0$$

$$0 \leq x_1$$

$$0 \leq x_2$$

START:	x_0	=	$(1 , .5)$	(feasible).
	$f(x_0)$	=	$-.01336459$	

SOLUTION:	x^*	=	$(3 , \sqrt{3})$
	$f(x^*)$	=	-1
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	2
	$I(x^*)$	=	$(1 , 3)$
	u^*_{max}/u^*_{min}	=	$.86603/.5 = 1.73$
	$\lambda^*_{max}/\lambda^*_{min}$	=	$-$

PROBLEM:	25
CLASSIFICATION:	SBR-T1-1
SOURCE:	Holzmann [32], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{i=1}^{99} (f_i(x))^2$$

$$f_i(x) = -.01i + \exp\left(-\frac{1}{x_1}(u_i - x_2)^{x_3}\right)$$

$$u_i = 25 + (-50 \ln(.01i))^{2/3}$$

$$i = 1,\ldots,99$$

CONSTRAINTS:

$$.1 \le x_1 \le 100$$

$$0 \le x_2 \le 25.6$$

$$0 \le x_3 \le 5$$

START: $x_o = (100 , 12.5 , 3)$ $f(x_o) = 32.835$ (feasible)

SOLUTION: $f(x^*) = 0$

$x^* = (50 , 25 , 1.5)$

$r(x^*) = 0$ $e(x^*) = -$

$\mu = 0$ $I(x^*) = -$

$u^*_{max}/u^*_{min} = -$

$\lambda^*_{max}/\lambda^*_{min} = 94.7/.14E-4 = .70E7$

PROBLEM:	26
CLASSIFICATION:	PPR-T1-1
SOURCE:	Huang, Aggerwal [34], Miele e.al. [43]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_2 - x_3)^4$$

CONSTRAINTS:

$$(1 + x_2{}^2)x_1 + x_3{}^4 - 3 = 0$$

START: $x_o = (-2.6 , 2 , 2)$ (feasible)

$f(x_o) = 21.16$

SOLUTION:

$x^* = (1 , 1 , 1) , (a , a , a)$

$f(x^*) = 0$

$r(x^*) = 0$ $\qquad a = \sqrt[3]{\alpha-\beta} - \sqrt[3]{\alpha+\beta} - 2/3$

$e(x^*) = 0$ $\qquad \alpha = \sqrt{139/108}$

$\mu = 0$ $\qquad \beta = 61/54$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0$

$\lambda^*_{max}/\lambda^*_{min} = 4/0$

PROBLEM:	27
CLASSIFICATION:	PQR-T1-6
SOURCE:	Miele e.al. [44,45]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = .01(x_1 - 1)^2 + (x_2 - x_1^2)^2$$

CONSTRAINTS:

$$x_1 + x_3^2 + 1 = 0$$

START: $x_o = (2, 2, 2)$ (not feasible)

$f(x_o) = 4.01$

SOLUTION: $x^* = (-1, 1, 0)$

$f(x^*) = .04$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .04/.04 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 2/.08 = 25$

PROBLEM:	28
CLASSIFICATION:	QLR-T1-2
SOURCE:	Huang, Aggerwal [34]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1(1)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 + x_2)^2 + (x_2 + x_3)^2$$

CONSTRAINTS:

$$x_1 + 2x_2 + 3x_3 - 1 = 0$$

START: $x_o = (-4 , 1 , 1)$ (feasible)

$f(x_o) = 13$

SOLUTION: $x^* = (.5 , -.5 , .5)$

$f(x^*) = 0$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0$

$\lambda^*_{max}/\lambda^*_{min} = 2.72/.42 = 6.45$

PROBLEM:	29
CLASSIFICATION:	PQR-T1-7
SOURCE:	Biggs [10]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = -x_1x_2x_3$$

CONSTRAINTS:

$$-x_1^2 - 2x_2^2 - 4x_3^2 + 48 \geq 0$$

START:

$x_o = (1 , 1 , 1)$ (feasible)

$f(x_o) = -1$

SOLUTION:

$x^* = (a,b,c) , (a,-b,-c) , (-a,b,-c) , (-a,-b,c)$

$f(x^*) = -16\sqrt{2}$

$r(x^*) = 0$

$e(x^*) = .19E-9$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .7071/.7071 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 7.79/3.52 = 2.22$

$a = 4$

$b = 2\sqrt{2}$

$c = 2$

PROBLEM:	30
CLASSIFICATION:	QQR-T1-8
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = x_1^2 + x_2^2 + x_3^2$$

CONSTRAINTS:

$$x_1^2 + x_2^2 - 1 \geq 0$$

$$1 \leq x_1 \leq 10$$

$$-10 \leq x_2 \leq 10$$

$$-10 \leq x_3 \leq 10$$

START: $x_0 = (1, 1, 1)$ (feasible)

$f(x_0) = 3$

SOLUTION:

$x^* = (1, 0, 0)$

$f(x^*) = 1$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 2$

$I(x^*) = (1, 2)$

$u^*_{max}/u^*_{min} = 1/0$

$\lambda^*_{max}/\lambda^*_{min} = 2/2 = 1$

PROBLEM:	31
CLASSIFICATION:	QQR-T1-9
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = 9x_1^2 + x_2^2 + 9x_3^2$$

CONSTRAINTS:

$$x_1 x_2 - 1 \geq 0$$

$$-10 \leq x_1 \leq 10$$

$$1 \leq x_2 \leq 10$$

$$-10 \leq x_3 \leq 1$$

START:

$x_o = (1 , 1 , 1)$ (feasible)

$f(x_o) = 19$

SOLUTION:

$x^* = (1/\sqrt{3} , \sqrt{3} , 0)$

$f(x^*) = 6$

$r(x^*) = 0$

$e(x^*) = .57E-10$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = 6/6 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 18/7.2 = 2.5$

PROBLEM:	32
CLASSIFICATION:	QPR-T1-2
SOURCE:	Evtushenko [25]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 1(1)$, $b = 3$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 + 3x_2 + x_3)^2 + 4(x_1 - x_2)^2$$

CONSTRAINTS:

$$6x_2 + 4x_3 - x_1^3 - 3 \geq 0$$

$$1 - x_1 - x_2 - x_3 = 0$$

$$0 \leq x_i \quad , \quad i = 1,2,3$$

START: $x_o = (.1 \ , \ .7 \ , \ .2)$ (feasible)

$f(x_o) = 7.2$

SOLUTION: $x^* = (0 \ , \ 0 \ , \ 1)$

$f(x^*) = 1$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 2$

$I(x^*) = (2 \ , \ 3)$

$u^*_{max}/u^*_{min} = 4/0$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	33
CLASSIFICATION:	PQR-T1-8
SOURCE:	Beltrami [6], Hartmann [28]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 1)(x_1 - 2)(x_1 - 3) + x_3$$

CONSTRAINTS:

$$x_3^2 - x_2^2 - x_1^2 \geq 0$$

$$x_1^2 + x_2^2 + x_3^2 - 4 \geq 0$$

$$0 \leq x_1$$

$$0 \leq x_2$$

$$0 \leq x_3 \leq 5$$

START: $x_0 = (0 , 0 , 3)$ (feasible)

$f(x_0) = -3$

SOLUTION: $x^* = (0 , \sqrt{2} , \sqrt{2})$

$f(x^*) = \sqrt{2} - 6$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 3$

$I(x^*) = (1 , 2 , 3)$

$u^*_{max}/u^*_{min} = 11/.17678 = 62.23$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	34
CLASSIFICATION:	LGR-T1-1
SOURCE:	Eckhardt [24]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = -x_1$$

CONSTRAINTS:

$$x_2 - \exp(x_1) \geq 0$$

$$x_3 - \exp(x_2) \geq 0$$

$$0 \leq x_1 \leq 100$$

$$0 \leq x_2 \leq 100$$

$$0 \leq x_3 \leq 10$$

START: $x_o = (0 , 1.05 , 2.9)$ (feasible)

$f(x_o) = 0$

SOLUTION:

$x^* = (\ln(\ln 10) , \ln 10 , 10)$

$f(x^*) = -\ln(\ln 10)$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 3$

$I(x^*) = (1 , 2 , 8)$

$u^*_{max}/u^*_{min} = .4343/.04343 = 10$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM: 35 (Beale's problem)

CLASSIFICATION: QLR-T1-3

SOURCE: Asaadi [1], Charalambous [18], Dimitru [23], Sheela [57]

NUMBER OF VARIABLES: $n = 3$

NUMBER OF CONSTRAINTS: $m_1 = 1(1)$, $m-m_1 = 0$, $b = 3$

OBJECTIVE FUNCTION:

$$f(x) = 9 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3$$

CONSTRAINTS:

$$3 - x_1 - x_2 - 2x_3 \geq 0$$

$$0 \leq x_i \quad , \quad i=1,2,3$$

START: $x_o = (.5 , .5 , .5)$ (feasible)

$f(x_o) = 2.25$

SOLUTION: $x^* = (4/3 , 7/9 , 4/9)$

$f(x^*) = 1/9$

$r(x^*) = 0$

$e(x^*) = .49E-10$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .2222/.2222 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 3.72/1.61 = 2.31$

PROBLEM:	36
CLASSIFICATION:	PLR-T1-2
SOURCE:	Biggs [10]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1(1)$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = -x_1x_2x_3$$

CONSTRAINTS:

$$72 - x_1 - 2x_2 - 2x_3 \geq 0$$

$$0 \leq x_1 \leq 20$$

$$0 \leq x_2 \leq 11$$

$$0 \leq x_3 \leq 42$$

START:	x_o	=	(10 , 10 , 10)	(feasible)
	$f(x_o)$	=	-1000	

SOLUTION:	x^*	=	(20 , 11 , 15)
	$f(x^*)$	=	-3300
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	3
	$I(x^*)$	=	(1 , 5 , 6)
	u^*_{max}/u^*_{min}	=	110/55 = 2
	$\lambda^*_{max}/\lambda^*_{min}$	=	-

PROBLEM:	37
CLASSIFICATION:	PLR-T1-3
SOURCE:	Betts [8], Box [12]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 2(2)$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = -x_1x_2x_3$$

CONSTRAINTS:

$$72 - x_1 - 2x_2 - 2x_3 \geq 0$$

$$x_1 + 2x_2 + 2x_3 \geq 0$$

$$0 \leq x_i \leq 42 \quad , \quad i=1,2,3$$

START: $x_o = (10\ ,\ 10\ ,\ 10)$ (feasible)

$f(x_o) = -1000$

SOLUTION: $x^* = (24\ ,\ 12\ ,\ 12)$

$f(x^*) = -3456$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = 144/144 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 24/8 = 3$

PROBLEM:	38 (Colville No.4)
CLASSIFICATION:	PBR-T1-4
SOURCE:	Colville [20], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 + 90(x_4 - x_3^2)^2 + (1-x_3)^2$$
$$+ 10.1((x_2 - 1)^2 + (x_4 - 1)^2) + 19.8(x_2 - 1)(x_4 - 1)$$

CONSTRAINTS:

$$-10 \le x_i \le 10 \quad , \quad i=1,\ldots,4$$

START: $x_o = (-3 , -1 , -3 , -1)$ (feasible)
$f(x_o) = 19192$

SOLUTION:
$x^* = (1 , 1 , 1 , 1)$
$f(x^*) = 0$
$r(x^*) = 0$
$e(x^*) = -$
$\mu = 0$
$I(x^*) = -$
$u^*_{max}/u^*_{min} = -$
$\lambda^*_{max}/\lambda^*_{min} = .10E4/.72 = .14E4$

PROBLEM:	39
CLASSIFICATION:	LPR-T1-1
SOURCE:	Miele e.al. [44,45]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = -x_1$$

CONSTRAINTS:

$$x_2 - x_1^3 - x_3^2 = 0$$

$$x_1^2 - x_2 - x_4^2 = 0$$

START:	x_0	=	$(2, 2, 2, 2)$	(not feasible)
	$f(x_0)$	=	-2	
SOLUTION:	x^*	=	$(1, 1, 0, 0)$	
	$f(x^*)$	=	-1	
	$r(x^*)$	=	0	
	$e(x^*)$	=	0	
	μ	=	0	
	$I(x^*)$	=	$-$	
	u^*_{max}/u^*_{min}	=	$1/1 = 1$	
	$\lambda^*_{max}/\lambda^*_{min}$	=	$2/2 = 1$	

PROBLEM:	40
CLASSIFICATION:	PPR-T1-2
SOURCE:	Beltrami [6], Indusi [35]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m - m_1 = 3$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = -x_1 x_2 x_3 x_4$$

CONSTRAINTS:

$$x_1^3 + x_2^2 - 1 = 0$$

$$x_1^2 x_4 - x_3 = 0$$

$$x_4^2 - x_2 = 0$$

START: $x_o = (.8\ ,\ .8\ ,\ .8\ ,\ .8)$ (not feasible)

$f(x_o) = -.4096$

SOLUTION: $x^* = (2^a\ ,\ 2^{2b}\ ,\ (-1)^i 2^c\ ,\ (-1)^i 2^b)$

$f(x^*) = -.25$ $\quad i = 1,2$

$r(x^*) = 0$ $\quad a = -1/3$

$e(x^*) = .80E-11$ $\quad b = -1/4$

$\mu = 0$ $\quad c = -11/12$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .5/.3536 = 1.41$

$\lambda^*_{max}/\lambda^*_{min} = 1.74/1.74 = 1$

PROBLEM:	41		
CLASSIFICATION:	PLR-T1-4		
SOURCE:	Betts [8], Miele e.al. [42]		
NUMBER OF VARIABLES:	$n = 4$		
NUMBER OF CONSTRAINTS:	$m_1 = 0$	, $m-m_1 = 1(1)$	, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = 2 - x_1 x_2 x_3$$

CONSTRAINTS:

$$x_1 + 2x_2 + 2x_3 - x_4 = 0$$

$$0 \le x_i \le 1 \quad , \quad i=1,2,3$$

$$0 \le x_4 \le 2$$

START:

$x_0 = (2\ ,\ 2\ ,\ 2\ ,\ 2)$ (not feasible)

$f(x_0) = -6$

SOLUTION:

$x^* = (2/3\ ,\ 1/3\ ,\ 1/3\ ,\ 2)$

$f(x^*) = 52/27$

$r(x^*) = 0$

$e(x^*) = .13E-10$

$\mu = 1$

$I(x^*) = (8)$

$u^*_{max}/u^*_{min} = .1111/.1111 = 1$

$\lambda^*_{max}/\lambda^*_{min} = .67/.22 = 3$

PROBLEM:	42
CLASSIFICATION:	QQR-T1-10
SOURCE:	Brusch [14]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2(1)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 1)^2 + (x_2 - 2)^2 + (x_3 - 3)^2 + (x_4 - 4)^2$$

CONSTRAINTS:

$$x_1 - 2 = 0$$

$$x_3^2 + x_4^2 - 2 = 0$$

START: $x_0 = (1 , 1 , 1 , 1)$ (not feasible)

$f(x_0) = 14$

SOLUTION: $x^* = (2 , 2 , .6\sqrt{2} , .8\sqrt{2})$

$f(x^*) = 28 - 10\sqrt{2}$

$r(x^*) = 0$

$e(x^*) = .2E-23$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 2.5355/2.0000 = 1.26$

$\lambda^*_{max}/\lambda^*_{min} = 7.07/2.00 = 3.54$

PROBLEM:	43 (Rosen-Suzuki)		
CLASSIFICATION:	QQR-T1-11		
SOURCE: Betts [8], Charalambous [18], Gould [27], Sheela [57]			
NUMBER OF VARIABLES:	$n = 4$		
NUMBER OF CONSTRAINTS:	$m_1 = 3$	, $m-m_1 = 0$	, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = x_1^2 + x_2^2 + 2x_3^2 + x_4^2 - 5x_1 - 5x_2 - 21x_3 + 7x_4$$

CONSTRAINTS:

$$8 - x_1^2 - x_2^2 - x_3^2 - x_4^2 - x_1 + x_2 - x_3 + x_4 \geq 0$$

$$10 - x_1^2 - 2x_2^2 - x_3^2 - 2x_4^2 + x_1 + x_4 \geq 0$$

$$5 - 2x_1^2 - x_2^2 - x_3^2 - 2x_1 + x_2 + x_4 \geq 0$$

START: $x_o = (0 , 0 , 0 , 0)$ (feasible)

$f(x_o) = 0$

SOLUTION: $x^* = (0 , 1 , 2 , -1)$

$f(x^*) = -44$

$r(x^*) = 0$

$e(x^*) = .21E-9$

$\mu = 2$

$I(x^*) = (1 , 3)$

$u^*_{max}/u^*_{min} = 2/1 = 2$

$\lambda^*_{max}/\lambda^*_{min} = 9/8.07 = 1.12$

PROBLEM:	44
CLASSIFICATION:	QLR-T1-4
SOURCE:	Konno [37]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 6(6)$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = x_1 - x_2 - x_3 - x_1x_3 + x_1x_4 + x_2x_3 - x_2x_4$$

CONSTRAINTS:

$$8 - x_1 - 2x_2 \geq 0$$

$$12 - 4x_1 - x_2 \geq 0$$

$$12 - 3x_1 - 4x_2 \geq 0$$

$$8 - 2x_3 - x_4 \geq 0$$

$$8 - x_3 - 2x_4 \geq 0$$

$$5 - x_3 - x_4 \geq 0 \quad , \quad 0 \leq x_i \quad , \quad i=1,\ldots,4$$

START: $x_o = (0 , 0 , 0 , 0)$ (feasible)

$f(x_o) = 0$

SOLUTION: $x^* = (0 , 3 , 0 , 4)$

$f(x^*) = -15$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 4$

$I(x^*) = (3 , 5 , 7 , 9)$

$u^*_{max}/u^*_{min} = 8.75/1.25 = 7$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	45	
CLASSIFICATION:	PBR-T1-5	
SOURCE:	Betts [8], Miele e.al. [42]	
NUMBER OF VARIABLES:	$n = 5$	
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$	, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = 2 - \frac{1}{120} x_1 x_2 x_3 x_4 x_5$$

CONSTRAINTS:

$$0 \le x_i \le i \quad , \quad i=1,\dots,5$$

START: $x_o = (2 , 2 , 2 , 2)$ (not feasible)

$f(x_o) = 26/15$

SOLUTION: $x^* = (1 , 2 , 3 , 4 , 5)$

$f(x^*) = 1$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 5$

$I(x^*) = (6 , 7 , 8 , 9 , 10)$

$u^*_{max}/u^*_{min} = 1/.2 = 5$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	47
CLASSIFICATION:	PPR-T1-3
SOURCE:	Huang, Aggerwal [34], Miele e.al. [43]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_2 - x_3)^3 + (x_3 - x_4)^4 + (x_4 - x_5)^4$$

CONSTRAINTS:

$$x_1 + x_2^2 + x_3^3 - 3 = 0$$

$$x_2 - x_3^2 + x_4 - 1 = 0$$

$$x_1 x_5 - 1 = 0$$

START: $x_o = (2 , \sqrt{2} , -1 , 2-\sqrt{2} , .5)$ (feasible)

$f(x_o) = 12.4954368$

SOLUTION: $x^* = (1 , 1 , 1 , 1 , 1)$

$f(x^*) = 0$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0/0$

$\lambda^*_{max}/\lambda^*_{min} = 2.08/.53 = 3.92$

PROBLEM:	48
CLASSIFICATION:	QLR-T1-5
SOURCE:	Huang, Aggerwal [34], Miele e.al. [43]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2(2)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 1)^2 + (x_2 - x_3)^2 + (x_4 - x_5)^2$$

CONSTRAINTS:

$$x_1 + x_2 + x_3 + x_4 + x_5 - 5 = 0$$

$$x_3 - 2(x_4 + x_5) + 3 = 0$$

START: $x_o = (3 , 5 , -3 , 2 , -2)$ (feasible)

$f(x_o) = 84$

SOLUTION: $x^* = (1 , 1 , 1 , 1 , 1)$

$f(x^*) = 0$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0/0$

$\lambda^*_{max}/\lambda^*_{min} = 4/1.49 = 2.69$

PROBLEM:	49
CLASSIFICATION:	PLR-T1-5
SOURCE:	Huang, Aggerwal [34]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2(2)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_3 - 1)^2 + (x_4 - 1)^4 + (x_5 - 1)^6$$

CONSTRAINTS:

$$x_1 + x_2 + x_3 + 4x_4 - 7 = 0$$

$$x_3 + 5x_5 - 6 = 0$$

START:	x_o	=	(10 , 7 , 2 , -3 , .8) (feasible)
	$f(x_o)$	=	266
SOLUTION:	x^*	=	(1 , 1 , 1 , 1 , 1)
	$f(x^*)$	=	0
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	0
	$I(x^*)$	=	-
	u^*_{max}/u^*_{min}	=	0/0
	$\lambda^*_{max}/\lambda^*_{min}$	=	4/.70E-10 = .57E11

PROBLEM:	50
CLASSIFICATION:	PLR-T1-6
SOURCE:	Huang, Aggerwal [34]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3(3)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_4)^4 + (x_4 - x_5)^2$$

CONSTRAINTS:

$$x_1 + 2x_2 + 3x_3 - 6 = 0$$

$$x_2 + 2x_3 + 3x_4 - 6 = 0$$

$$x_3 + 2x_4 + 3x_5 - 6 = 0$$

START: $x_o = (35 , -31 , 11 , 5 , -5)$ (feasible)

$f(x_o) = 17416$

SOLUTION: $x^* = (1 , 1 , 1 , 1 , 1)$

$f(x^*) = 0$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0/0$

$\lambda^*_{max}/\lambda^*_{min} = 5.89/1.64 = 3.6$

PROBLEM:	51
CLASSIFICATION:	QLR-T1-6
SOURCE:	Huang, Aggerwal [34]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m - m_1 = 3(3)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_2 + x_3 - 2)^2 + (x_4 - 1)^2 + (x_5 - 1)^2$$

CONSTRAINTS:

$$x_1 + 3x_2 - 4 = 0$$

$$x_3 + x_4 - 2x_5 = 0$$

$$x_2 - x_5 = 0$$

START: $x_o = (2.5 , .5 , 2 , -1 , .5)$ (feasible)

$f(x_o) = 8.5$

SOLUTION: $x^* = (1 , 1 , 1 , 1 , 1)$

$f(x^*) = 0$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 0/0$

$\lambda^*_{max}/\lambda^*_{min} = 3.49/1.90 = 1.84$

PROBLEM:	52
CLASSIFICATION:	QLR-T1-7
SOURCE:	Miele e.al. [44,45]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3(3)$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (4x_1 - x_2)^2 + (x_2 + x_3 - 2)^2 + (x_4 - 1)^2 + (x_5 - 1)^2$$

CONSTRAINTS:

$$x_1 + 3x_2 = 0$$

$$x_3 + x_4 - 2x_5 = 0$$

$$x_2 - x_5 = 0$$

START: $x_o = (2 , 2 , 2 , 2 , 2)$ (not feasible)

$f(x_o) = 42$

SOLUTION: $x^* = (-33 , 11 , 180 , -158 , 11)/349$

$f(x^*) = 1859/349$

$r(x^*) = 0$

$e(x^*) = .14E-9$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 7.7479/2.9054 = 2.6667$

$\lambda^*_{max}/\lambda^*_{min} = 26.93/1.99 = 13.51$

PROBLEM:	53
CLASSIFICATION:	QLR-T1-8
SOURCE:	Betts [8], Miele e.al. [42,43]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3(3)$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_2 + x_3 - 2)^2 + (x_4 - 1)^2 + (x_5 - 1)^2$$

CONSTRAINTS:

$$x_1 + 3x_2 = 0$$

$$x_3 + x_4 - 2x_5 = 0$$

$$x_2 - x_5 = 0$$

$$-10 \le x_i \le 10 \quad , \quad i=1,\ldots,5$$

START:

$x_o = (2 , 2 , 2 , 2 , 2)$ (not feasible)

$f(x_o) = 6$

SOLUTION:

$x^* = (-33 , 11 , 27 , -5 , 11)/43$

$f(x^*) = 176/43$

$r(x^*) = 0$

$e(x^*) = .28E-9$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 5.9535/2.0465 = 1.84$

$\lambda^*_{max}/\lambda^*_{min} = 3.49/1.90 = 1.84$

PROBLEM:	54
CLASSIFICATION:	GLR-T1-2
SOURCE:	Betts [8], Picket [50]
NUMBER OF VARIABLES:	$n = 6$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1(1)$, $b = 12$

OBJECTIVE FUNCTION:

$$f(x) = -\exp(-h(x)/2)$$

$$\begin{aligned} h(x) = \; & ((x_1 - 1.E6)^2/6.4E7 + (x_1 - 1.E4)(x_2 - 1)/2.E4 \\ & + (x_2 - 1)^2)(x_3 - 2.E6)^2/(.96\cdot 4.9E13) \\ & + (x_4 - 10)^2/2.5E3 + (x_5 - 1.E-3)^2/2.5E-3 \\ & + (x_6 - 1.E8)^2/2.5E17 \end{aligned}$$

CONSTRAINTS:

$$x_1 + 4.E3x_2 - 1.76E4 = 0$$

$$0 \le x_1 \le 2.E4 \qquad -10 \le x_2 \le 10 \qquad 0 \le x_3 \le 1.E7$$

$$0 \le x_4 \le 20 \qquad -1 \le x_5 \le 1 \qquad 0 \le x_6 \le 2.E8$$

START: x_0 = (6E3 , 1.5 , 4E6 , 2 , 3E-3 , 5E7)

$f(x_0)$ = -.7651 (not feasible)

SOLUTION: x^* = (91600/7,79/70,2E6,10,1E-3,1E8)

$f(x^*)$ = -exp(-27/280)

$r(x^*)$ = 0

$e(x^*)$ = .20E-10

μ = 0

$I(x^*)$ = -

u^*_{max}/u^*_{min} = .4865E-4/.4865E-4 = 1

$\lambda^*_{max}/\lambda^*_{min}$ = 362.9/.36E-17 = .10E21

PROBLEM:	55
CLASSIFICATION:	GLR-T1-3
SOURCE:	Hsia [33]
NUMBER OF VARIABLES:	$n = 6$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 6(6)$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = x_1 + 2x_2 + 4x_5 + \exp(x_1 x_4)$$

CONSTRAINTS:

$$x_1 + 2x_2 + 5x_5 - 6 = 0$$

$$x_1 + x_2 + x_3 - 3 = 0$$

$$x_4 + x_5 + x_6 - 2 = 0$$

$$x_1 + x_4 - 1 = 0$$

$$x_2 + x_5 - 2 = 0$$

$$x_3 + x_6 - 2 = 0$$

$$0 \le x_i \quad , \quad i=1,\ldots,6 \quad , \quad x_1 \le 1 \quad , \quad x_4 \le 1$$

START: $x_o = (1 , 2 , 0 , 0 , 0 , 2)$ (not feasible)

$f(x_o) = 6$

SOLUTION: $x^* = (0 , 4/3 , 5/3 , 1 , 2/3 , 1/3)$

$f(x^*) = 19/3$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 8$

$I(x^*) = (1 , 8)$

$u^*_{max}/u^*_{min} = -$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	56
CLASSIFICATION:	PGR-T1-2
SOURCE:	Brusch [15]
NUMBER OF VARIABLES:	$n = 7$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 4$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = - x_1 x_2 x_3$$

CONSTRAINTS:

$$x_1 - 4.2\sin^2 x_4 = 0$$

$$x_2 - 4.2\sin^2 x_5 = 0$$

$$x_3 - 4.2\sin^2 x_6 = 0$$

$$x_1 + 2x_2 + 2x_3 - 7.2\sin^2 x_7 = 0$$

START: $x_o = (1 , 1 , 1 , a , a , a , b)$

$f(x_o) = -1$ (feasible)

SOLUTION:

$x^* = (2.4, 1.2, 1.2, \pm c + j\pi, \pm d + k\pi, \pm d + l\pi, (r+.5)\pi)$

$f(x^*) = -3.456$ $\quad a = \arcsin\sqrt{1/4.2}$

$r(x^*) = 0$ $\quad b = \arcsin\sqrt{5/7.2}$

$e(x^*) = .67E-10$ $\quad c = \arcsin\sqrt{4/7}$

$\mu = 0$ $\quad d = \arcsin\sqrt{2/7}$

$I(x^*) = -$ $\quad j,k,l,r = 0, \pm 1, \pm 2, \ldots$

$u^*_{max}/u^*_{min} = 1.44/.68E-11 = .21E12$

$\lambda^*_{max}/\lambda^*_{min} = 20.74/.76 = 27.45$

PROBLEM:	57
CLASSIFICATION:	SQR-P1-1
SOURCE:	Betts [8], Gould [27]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 2$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{i=1}^{44} f_i(x)^2$$

$$f_i(x) = b_i - x_1 - (.49 - x_1)\exp(-x_2(a_i - 8))$$

$$i=1,\ldots,44$$

a_i, b_i: cf. Appendix A

CONSTRAINTS:

$$.49x_2 - x_1x_2 - .09 \geq 0$$

$$.4 \leq x_1 \quad , \quad -4 \leq x_2$$

START:	x_o	=	(.42 , 5) (feasible)
	$f(x_o)$	=	.030798602
SOLUTION:	x^*	=	(.419952675 , 1.284845629)
	$f(x^*)$	=	.02845966972
	$r(x^*)$	=	0
	$e(x^*)$	=	.98E-7
	μ	=	1
	$I(x^*)$	=	(1)
	u^*_{max}/u^*_{min}	=	.06671/.06671 = 1
	$\lambda^*_{max}/\lambda^*_{min}$	=	.23/.23 = 1

PROBLEM:	59
CLASSIFICATION:	GQR-P1-1
SOURCE:	Barnes [3], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 2$
NUMBER OF CONSTRAINTS:	$m_1 = 3$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$\begin{aligned} f(x) = & -75.196 + 3.8112x_1 + .0020567x_1^3 - 1.0345E{-}5x_1^4 \\ & + 6.8306x_2 - .030234x_1x_2 + 1.28134E{-}3x_2x_1^2 \\ & + 2.266E{-}7x_1^4x_2 - .25645x_2^2 + .0034604x_2^3 - 1.3514E{-}5x_2^4 \\ & + 28.106/(x_2 + 1) + 5.2375E{-}6x_1^2x_2^2 + 6.3E{-}8x_1^3x_2^2 \\ & - 7E{-}10x_1^3x_2^3 - 3.405E{-}4x_1x_2^2 + 1.6638E{-}6x_1x_2^3 \\ & + 2.8673\exp(.0005x_1x_2) - 3.5256E{-}5x_1^3x_2 \end{aligned}$$

CONSTRAINTS:

$$x_1x_2 - 700 \ge 0$$

$$x_2 - x_1^2/125 \ge 0 \qquad 0 \le x_1 \le 75$$

$$(x_2 - 50)^2 - 5(x_1 - 55) \ge 0 \qquad 0 \le x_2 \le 65$$

START: $x_o = (90 , 10)$ (not feasible)

$f(x_o) = 86.878639$

SOLUTION: $x^* = (13.55010424 , 51.66018129)$

$f(x^*) = -7.804226324$

$r(x^*) = 0$

$e(x^*) = .27E{-}6$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .01142/.01142 = 1$

$\lambda^*_{max}/\lambda^*_{min} = .13/.13 = 1$

PROBLEM:	60
CLASSIFICATION:	PPR-P1-1
SOURCE:	Betts [8], Miele e.al. [42,44]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 1)^2 + (x_1 - x_2)^2 + (x_2 - x_3)^4$$

CONSTRAINTS:

$$x_1(1 + x_2^2) + x_3^4 - 4 - 3\sqrt{2} = 0$$

$$-10 \le x_i \le 10 \quad , \quad i=1,2,3$$

START:	x_o	=	(2 , 2 , 2) (not feasible)
	$f(x_o)$	=	1
SOLUTION:	x^*	=	(1.104859024,1.196674194,1,535262257)
	$f(x^*)$	=	.03256820025
	$r(x^*)$	=	.23E-9
	$e(x^*)$	=	.38E-7
	μ	=	1
	$I(x^*)$	=	(1)
	u^*_{max}/u^*_{min}	=	.01073/.01073 = 1
	$\lambda^*_{max}/\lambda^*_{min}$	=	5.72/2.07 = 2.76

PROBLEM:	61		
CLASSIFICATION:	QQR-P1-1		
SOURCE:	Fletcher, Lill [26]		
NUMBER OF VARIABLES:	$n = 3$		
NUMBER OF CONSTRAINTS:	$m_1 = 0$	, $m-m_1 = 2$	, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = 4x_1^2 + 2x_2^2 + 2x_3^2 - 33x_1 + 16x_2 - 24x_3$$

CONSTRAINTS:

$$3x_1 - 2x_2^2 - 7 = 0$$

$$4x_1 - x_3^2 - 11 = 0$$

START: $x_o = (0 , 0 , 0)$ (not feasible)

$f(x_o) = 0$

SOLUTION: $x^* = (5.326770157,-2.118998639,3.210464239)$

$f(x^*) = -143.6461422$

$r(x^*) = .29E-9$

$e(x^*) = .21E-6$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = 1.7378/.8877 = 1.96$

$\lambda^*_{max}/\lambda^*_{min} = 7.83/7.83 = 1$

PROBLEM:	62
CLASSIFICATION:	GLR-P1-1
SOURCE:	Betts [8], Picket [50]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 1(1)$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = -32.174(255\ \ln((x_1+x_2+x_3+.03)/(.09x_1+x_2+x_3+.03))$$
$$+ 280\ \ln((x_2+x_3+.03)/(.07x_2+x_3+.03))$$
$$+ 290\ \ln((x_3+.03)/(.13x_3+.03)))$$

CONSTRAINTS:

$$x_1 + x_2 + x_3 - 1 = 0$$
$$0 \le x_i \le 1 \quad , \quad i=1,2,3$$

START:	x_o	=	(.7 , .2 , .1) (feasible)
	$f(x_o)$	=	-25698.3
SOLUTION:	x^*	=	(.6178126908,.328202223,.5398508606E-1)
	$f(x^*)$	=	-26272.51448
	$r(x^*)$	=	0
	$e(x^*)$	=	.20E-5
	μ	=	0
	$I(x^*)$	=	-
	u^*_{max}/u^*_{min}	=	6387/6387 = 1
	$\lambda^*_{max}/\lambda^*_{min}$	=	.32E6/.72E4 = 44.9

PROBLEM:	63
CLASSIFICATION:	QQR-P1-2
SOURCE:	Himmelblau [29], Paviani [48], Sheela [57]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2(1)$, $b = 3$

OBJECTIVE FUNCTION:

$$f(x) = 1000 - x_1^2 - 2x_2^2 - x_3^2 - x_1x_2 - x_1x_3$$

CONSTRAINTS:

$$8x_1 + 14x_2 + 7x_3 - 56 = 0$$

$$x_1^2 + x_2^2 + x_3^2 - 25 = 0$$

$$0 \le x_i \quad , \quad i=1,2,3$$

START:	x_o	=	(2 , 2 , 2) (not feasible)
	$f(x_o)$	=	976
SOLUTION:	x^*	=	(3.512118414,.2169881741,3.552174034)
	$f(x^*)$	=	961.7151721
	$r(x^*)$	=	0
	$e(x^*)$	=	.62E-5
	μ	=	0
	$I(x^*)$	=	-
	u^*_{max}/u^*_{min}	=	1.223/.2749 = 4.45
	$\lambda^*_{max}/\lambda^*_{min}$	=	1.52/1.52 = 1

PROBLEM:	64
CLASSIFICATION:	PPR-P1-2
SOURCE:	Best [7]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 3$

OBJECTIVE FUNCTION:

$$f(x) = 5x_1 + 50000/x_1 + 20x_2 + 72000/x_2 + 10x_3 + 144000/x_3$$

CONSTRAINTS:

$$1 - 4/x_1 - 32/x_2 - 120/x_3 \geq 0$$

$$1.E-5 \leq x_i \quad , \quad i=1,2,3$$

START:	x_o	=	(1 , 1 , 1) (not feasible)
	$f(x_o)$	=	266035
SOLUTION:	x^*	=	(108.7347175,85.12613942,204.3247078)
	$f(x^*)$	=	6299.842428
	$r(x^*)$	=	0
	$e(x^*)$	=	.28E-4
	μ	=	1
	$I(x^*)$	=	(1)
	u^*_{max}/u^*_{min}	=	2279/2279 = 1
	$\lambda^*_{max}/\lambda^*_{min}$	=	.21/.092 = 2.28

PROBLEM:	65
CLASSIFICATION:	QQR-P1-3
SOURCE:	Murtagh, Sargent [47]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - x_2)^2 + (x_1 + x_2 - 10)^2/9 + (x_3 - 5)^2$$

CONSTRAINTS:

$$48 - x_1^2 - x_2^2 - x_3^2 \geq 0$$

$$-4.5 \leq x_i \leq 4.5 \quad , \quad i=1,2$$

$$-5 \leq x_3 \leq 5$$

START:

$x_o = (-5 , 5 , 0)$ (not feasible)

$f(x_o) = 1225/9$

SOLUTION:

$x^* = (3.650461821,3.65046168,4.6204170507)$

$f(x^*) = .9535288567$

$r(x^*) = 0$

$e(x^*) = .40E-6$

$\mu = 1$

$I(x^*) = (1)$

$u^*_{max}/u^*_{min} = .08215/.08215 = 1$

$\lambda^*_{max}/\lambda^*_{min} = 1.95/1.68 = 1.16$

PROBLEM:	66
CLASSIFICATION:	LGR-P1-1
SOURCE:	Eckhardt [24]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = .2x_3 - .8x_1$$

CONSTRAINTS:

$$x_2 - \exp(x_1) \geq 0$$

$$x_3 - \exp(x_2) \geq 0$$

$$0 \leq x_1 \leq 100$$

$$0 \leq x_2 \leq 100$$

$$0 \leq x_3 \leq 10$$

START:	x_o	=	(0 , 1.05 , 2.9) (feasible)
	$f(x_o)$	=	.58
SOLUTION:	x^*	=	(.1841264879,1.202167873,3.327322322)
	$f(x^*)$	=	.5181632741
	$r(x^*)$	=	.58E-10
	$e(x^*)$	=	.86E-11
	μ	=	2
	$I(x^*)$	=	(1 , 2)
	u^*_{max}/u^*_{min}	=	.6654/.2 = 3.33
	$\lambda^*_{max}/\lambda^*_{min}$	=	.096/.096 = 1

PROBLEM:	67 (Colville No.8)
CLASSIFICATION:	GGI-P1-1
SOURCE:	Colville [20], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 3$
NUMBER OF CONSTRAINTS:	$m_1 = 14$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = -(.063y_2(x)y_5(x) - 5.04x_1 - 3.36y_3(x) - .035x_2 - 10x_3)$$

$y_i(x)$: cf. Appendix A

CONSTRAINTS:

$$y_{i+1}(x) - a_i \geq 0 \quad , \quad i=1,\ldots,7$$

$$a_i - y_{i-6}(x) \geq 0 \quad , \quad i=8,\ldots,14$$

$$1.E-5 \leq x_1 \leq 2.E3$$

$$1.E-5 \leq x_2 \leq 1.6E4$$

$$1.E-5 \leq x_3 \leq 1.2E2$$

a_i : cf. Appendix A

START: $x_o = (1745 \ , \ 12000 \ , \ 110)$ (feasible)

$f(x_o) = 868.6458$

SOLUTION:

$x^* = (1728.371286, 16000.00000, 98.14151402)$

$f(x^*) = -1162.036507$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 3$

$I(x^*) = (9 \ , \ 11 \ , \ 19)$

$u^*_{max}/u^*_{min} = 1.5872/.03403 = 46.6$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	68 , 69 (cost optimal inspection plan)		
CLASSIFICATION:	GGR-P1-(1,2)		
SOURCE:	Collani [19]		
NUMBER OF VARIABLES:	$n = 4$		
NUMBER OF CONSTRAINTS:	$m_1 = 0$	$m-m_1 = 2$	$b = 8$

OBJECTIVE FUNCTION:

$$f(x) = (a_i n_i - \frac{b_i(\exp(x_1) - 1) - x_3}{\exp(x_1) - 1 + x_4} x_4)/x_1 \quad , \quad i=1,2$$

No. 68 : $a_1 = .0001$, $b_1 = 1$, $d_1 = 1$, $n_1 = 24$

No. 69 : $a_2 = .1$, $b_2 = 1000$, $d_2 = 1$, $n_2 = 4$

CONSTRAINTS:

$$x_3 - 2\Phi(-x_2) = 0$$

$$x_4 - \Phi(-x_2 + d_i\sqrt{n_i}) - \Phi(-x_2 - d_i\sqrt{n_i}) = 0$$

$$\Phi(x) = \int_{-\infty}^{x} \exp(-y^2/2)/\sqrt{2\pi} \; dy$$

$.0001 \le x_1 \le 100$, $0 \le x_2 \le 100$, $0 \le x_3 \le 2$, $0 \le x_4 \le 2$

START:	x_0	=	(1 , 1 , 1 , 1) (not feasible) – for both problems	
	$f(x_0)$	=	-.2618407	-631.3525
SOLUTION:	x^*	=	(.06785874,3.6461717,.00026617,.8948622)	
			(.02937141,1.1902534,.23394676,.7916678)	
	$f(x^*)$	=	-.920425026	-956.71288
	$r(x^*)$	=	.54E-7	.44E-10
	$e(x^*)$	=	.14E-4	.33E-4
	μ	=	0	0
	$I(x^*)$	=	–	–
	u^*_{max}/u^*_{min}	=	13.66/.0777 = 176	44.47/32.81 = 1.3
	$\lambda^*_{max}/\lambda^*_{min}$	=	16.4/.062 = .26E3	.26E5/19.6 = .1E4

PROBLEM:	70
CLASSIFICATION:	SQR-P1-1
SOURCE:	Himmelblau [29,30]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{i=1}^{19} (y_{i,cal} - y_{i,obs})^2$$

$$y_{i,cal} = (1 + \frac{1}{12x_2})[x_3 b^{x_2} (x_2/6.2832)^{.5} (c_i/7.685)^{x_2-1}$$

$$\exp(x_2 - bc_i x_2/7.658)] + (1 + \frac{1}{12x_1})[(1 - x_3)(b/x_4)^{x_1}$$

$$(x_1/6.2832)^{.5} (c_i/7.658)^{x_1-1} \exp(x_1 - bc_i x_1/(7.658x_4))]$$

$$b = x_3 + (1 - x_3)x_4$$

c_i, $y_{i,obs}$: cf. Appendix A

CONSTRAINTS:

$$x_3 + (1 - x_3)x_4 \geq 0$$

$$.00001 \leq x_i \leq 100 \quad , \quad i=1,2,4$$

$$.00001 \leq x_3 \leq 1$$

START:	x_o	=	(2 , 4 , .04 , 2) (feasible)
	$f(x_o)$	=	.9818596
SOLUTION:	x^*	=	(12.27695,4.631788,.3128625,2.029290)
	$f(x^*)$	=	.007498464
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	0
	$I(x^*)$	=	-
	u^*_{max}/u^*_{min}	=	-
	$\lambda^*_{max}/\lambda^*_{min}$	=	18.1/.91E-3 = .20E5

PROBLEM:	71
CLASSIFICATION:	PPR-P1-3
SOURCE:	Bartholomew-Biggs [4]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 1$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = x_1x_4(x_1 + x_2 + x_3) + x_3$$

CONSTRAINTS:

$$x_1x_2x_3x_4 - 25 \geq 0$$

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 - 40 = 0$$

$$1 \leq x_i \leq 5 \quad , \quad i=1,\ldots,4$$

START:	x_o	=	(1 , 5 , 5 , 1) (feasible)
	$f(x_o)$	=	16
SOLUTION:	x^*	=	(1 , 4.7429994 , 3.8211503 , 1.3794082)
	$f(x^*)$	=	17.0140173
	$r(x^*)$	=	0
	$e(x^*)$	=	.51E-6
	μ	=	2
	$I(x^*)$	=	(1 , 2)
	u^*_{max}/u^*_{min}	=	1.0879/.1615 = 6.74
	$\lambda^*_{max}/\lambda^*_{min}$	=	1.18/1.18 = 1

PROBLEM:	72 (optimal sample size)
CLASSIFICATION:	LPR-P1-1
SOURCE:	Bracken, McCormick [13]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = 1 + x_1 + x_2 + x_3 + x_4$$

CONSTRAINTS:

$$.0401 - 4/x_1 - 2.25/x_2 - 1/x_3 - .25/x_4 \geq 0$$

$$.010085 - .16/x_1 - .36/x_2 - .64/x_3 - .64/x_4 \geq 0$$

$$.001 \leq x_i \leq (5 - i)E5 \quad , \quad i=1,\ldots,4$$

START: $x_0 = (1 , 1 , 1 , 1)$ (not feasible)

$f(x_0) = 5$

SOLUTION:

$x^* = (193.4071, 179.5475, 185.0186, 168.7062)$

$f(x^*) = 727.67937$

$r(x^*) = 0$

$e(x^*) = .11E-4$

$\mu = 2$

$I(x^*) = (1 , 2)$

$u^*_{max}/u^*_{min} = .4147E5/.7693E4 = 5.39$

$\lambda^*_{max}/\lambda^*_{min} = .011/.011 = 1.02$

PROBLEM:	73 (cattle-feed)
CLASSIFICATION:	LGI-P1-1
SOURCE:	Biggs [10], Bracken, McCormick [13]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 2(1)$, $m-m_1 = 1(1)$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = 24.55x_1 + 26.75x_2 + 39x_3 + 40.50x_4$$

CONSTRAINTS:

$$2.3x_1 + 5.6x_2 + 11.1x_3 + 1.3x_4 - 5 \geq 0$$

$$12x_1 + 11.9x_2 + 41.8x_3 + 52.1x_4 - 21$$
$$- 1.645(.28x_1^2 + .19x_2^2 + 20.5x_3^2 + .62x_4^2)^{1/2} \geq 0$$

$$x_1 + x_2 + x_3 + x_4 - 1 = 0$$

$$0 \leq x_i \quad , \quad i=1,\ldots,4$$

START: x_o = (1 , 1 , 1 , 1) (not feasible)
$f(x_o)$ = 130.8

SOLUTION: x^* = (.6355216,-.12E-11,.3127019,.05177655)
$f(x^*)$ = 29.894378
$r(x^*)$ = .99E-10
$e(x^*)$ = 0
μ = 3
$I(x^*)$ = (1 , 2 , 4)
u^*_{max}/u^*_{min} = 18.37/.2433 = 75.5
$\lambda^*_{max}/\lambda^*_{min}$ = -

PROBLEM:	74 , 75
CLASSIFICATION:	PGR-P1-(1,2)
SOURCE:	Beuneu [9]
NUMBER OF VARIABLES:	n = 4
NUMBER OF CONSTRAINTS:	$m_1 = 2(2)$, $m-m_1 = 3$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = 3x_1 + 1.E\text{-}6x_1^3 + 2x_2 + \tfrac{2}{3}E\text{-}6x_2^3$$

CONSTRAINTS:

$$x_4 - x_3 + a_j \geq 0$$

$$x_3 - x_4 + a_j \geq 0$$

$$1000\sin(-x_3 - .25) + 1000\sin(-x_4 - .25) + 894.8 - x_1 = 0$$

$$1000\sin(x_3 - .25) + 1000\sin(x_3 - x_4 - .25) + 894.8 - x_2 = 0$$

$$1000\sin(x_4 - .25) + 1000\sin(x_4 - x_3 - .25) + 1294.8 = 0$$

$$0 \leq x_i \leq 1200 \quad , \quad i=1,2$$

$$-a_j \leq x_i \leq a_j \quad , \quad i=3,4$$

No. 74 : $a_1 = .55$ No.75 : $a_2 = .48$

START: x_o = (0 , 0 , 0 , 0) (not feasible) – for both problems

$f(x_o)$ = 0

SOLUTION:

		No. 74	No. 75
x^*	=	(679.9453,1026.067,.1188764,-.3962336)	(776.1592,925.1949,.05110879, -.4288911)
$f(x^*)$	=	5126.4981	5174.4129
$r(x^*)$	=	.75E-7	.30E-7
$e(x^*)$	=	.52E-7	0
μ	=	0	1
$I(x^*)$	=	–	(1)
u^*_{max}/u^*_{min}	=	5.46/4.11 = 1.33	2779/3.712 = 748.7
$\lambda^*_{max}/\lambda^*_{min}$	=	.49E-2/.49E-2 = 1	–

PROBLEM:	76
CLASSIFICATION:	QLR-P1-1
SOURCE:	Murtagh, Sargent [47]
NUMBER OF VARIABLES:	$n = 4$
NUMBER OF CONSTRAINTS:	$m_1 = 3(3)$, $m-m_1 = 0$, $b = 4$

OBJECTIVE FUNCTION:

$$f(x) = x_1^2 + .5x_2^2 + x_3^2 + .5x_4^2 - x_1x_3 + x_3x_4$$
$$- x_1 - 3x_2 + x_3 - x_4$$

CONSTRAINTS:

$$5 - x_1 - 2x_2 - x_3 - x_4 \geq 0$$

$$4 - 3x_1 - x_2 - 2x_3 + x_4 \geq 0$$

$$x_2 + 4x_3 - 1.5 \geq 0$$

$$0 \leq x_i \quad , \quad i=1,\ldots,4$$

START: $x_o = (.5 , .5 , .5 , .5)$ (feasible)

$f(x_o) = -1.25$

SOLUTION: $x^* = (.2727273, 2.090909, -.26E-10, .5454545)$

$f(x^*) = -4.681818181$

$r(x^*) = .84E-10$

$e(x^*) = .15E-10$

$\mu = 2$

$I(x^*) = (1 , 6)$

$u^*_{max}/u^*_{min} = 1.7272/.4545 = 3.8$

$\lambda^*_{max}/\lambda^*_{min} = 1.83/1 = 1.83$

PROBLEM:	77
CLASSIFICATION:	PGR-P1-3
SOURCE:	Betts [8], Miele e.al. [42,44,45]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 1)^2 + (x_1 - x_2)^2 + (x_3 - 1)^2 + (x_4 - 1)^4 + (x_5 - 1)^6$$

CONSTRAINTS:

$$x_1^2 x_4 + \sin(x_4 - x_5) - 2\sqrt{2} = 0$$

$$x_2 + x_3^4 x_4^2 - 8 - \sqrt{2} = 0$$

START: $x_0 = (2\ ,\ 2\ ,\ 2\ ,\ 2\ ,\ 2\ ,\ 2)$

$f(x_0) = 4$ (not feasible)

SOLUTION:

$x^* = (1.166172, 1.182111, 1.380257, 1.506036, .6109203)$

$f(x^*) = .24150513$

$r(x^*) = .12E-9$

$e(x^*) = .53E-7$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .08554/.03188 = 2.68$

$\lambda^*_{max}/\lambda^*_{min} = 3.92/.75 = 5.25$

PROBLEM:	78
CLASSIFICATION:	PPR-P1-4
SOURCE:	Asaadi [1], Powell [51]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = x_1x_2x_3x_4x_5$$

CONSTRAINTS:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0$$

$$x_2x_3 - 5x_4x_5 = 0$$

$$x_1^3 + x_2^3 + 1 = 0$$

START: $x_o = (-2 , 1.5 , 2 , -1 , -1)$

$f(x_o) = -6$ (not feasible)

SOLUTION:

$x^* = (-1.717142, 1.595708, 1.827248, -.7636429, -.7636435)$

$f(x^*) = -2.91970041$

$r(x^*) = .35E-9$

$e(x^*) = .91E-5$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .7444/.09681 = 7.69$

$\lambda^*_{max}/\lambda^*_{min} = 3.04/2.98 = 1.02$

PROBLEM:	79
CLASSIFICATION:	PPR-P1-5
SOURCE:	Betts [8], Miele e.al. [42,44,45]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 1)^2 + (x_1 - x_2)^2 + (x_2 - x_3)^2 + (x_3 - x_4)^4 + (x_4 - x_5)^4$$

CONSTRAINTS:

$$x_1 + x_2^2 + x_3^3 - 2 - 3\sqrt{2} = 0$$

$$x_2 - x_3^2 + x_4 + 2 - 2\sqrt{2} = 0$$

$$x_1 x_5 - 2 = 0$$

START: $x_0 = (2 , 2 , 2 , 2 , 2)$ (not feasible)

$f(x_0) = 1$

SOLUTION:

$x^* = (1.191127, 1.362603, 1.472818, 1.635017, 1.679081)$

$f(x^*) = .0787768209$

$r(x^*) = .58E-9$

$e(x^*) = .71E-10$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .3882E-1/.2873E-3 = 135.1$

$\lambda^*_{max}/\lambda^*_{min} = 2.03/.70 = 2.88$

PROBLEM:	80
CLASSIFICATION:	GPR-P1-1
SOURCE:	Powell [52]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = \exp(x_1 x_2 x_3 x_4 x_5)$$

CONSTRAINTS:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0$$

$$x_2 x_3 - 5 x_4 x_5 = 0$$

$$x_1^3 + x_2^3 + 1 = 0$$

$$-2.3 \le x_i \le 2.3 \quad , \quad i=1,2$$

$$-3.2 \le x_i \le 3.2 \quad , \quad i=3,4,5$$

START: $x_o = (-2 , 2 , 2 , -1 , -1)$ (not feasible)

$f(x_o) = 3.3546E-4$

SOLUTION: $x^* = (-1.717143, 1.595709, 1.827247, -.7636413, -.7636450)$

$f(x^*) = .0539498478$

$r(x^*) = .41E-9$

$e(x^*) = .49E-6$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = .04016/.005222 = 7.69$

$\lambda^*_{max}/\lambda^*_{min} = .16/.16 = 1.02$

PROBLEM:	81		
CLASSIFICATION:	GPR-P1-2		
SOURCE:	Powell [52]		
NUMBER OF VARIABLES:	$n = 5$		
NUMBER OF CONSTRAINTS:	$m_1 = 0$	, $m-m_1 = 3$	, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = \exp(x_1x_2x_3x_4x_5) - .5(x_1^3 + x_2^3 + 1)^2$$

CONSTRAINTS:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 - 10 = 0$$

$$x_2x_3 - 5x_4x_5 = 0$$

$$x_1^3 + x_2^3 + 1 = 0$$

$$-2.3 \le x_i \le 2.3 \quad , \quad i=1,2$$

$$-3.2 \le x_i \le 3.2 \quad , \quad i=3,4,5$$

START:	x_o	=	(-2 , 2 , 2 , -1 , -1) (not feasible)
	$f(x_o)$	=	-.49966
SOLUTION:	x^*	=	(-1.717142,1.159571,1.827248,-.7636474, -.7636390)
	$f(x^*)$	=	.0539498478
	$r(x^*)$	=	.21E-9
	$e(x^*)$	=	.11E-5
	μ	=	0
	$I(x^*)$	=	-
	u^*_{max}/u^*_{min}	=	.04016/.005223 = 7.69
	$\lambda^*_{max}/\lambda^*_{min}$	=	.16/.16 = 1.02

PROBLEM:	83 (Colville No.3)
CLASSIFICATION:	QQR-P1-4
SOURCE:	Colville [20], Dembo [22], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 6$, $m-m_1 = 0$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = 5.3578547x_3^2 + .8356891x_1x_5 + 37.293239x_1 - 40792.141$$

CONSTRAINTS:

$$92 \geq a_1 + a_2x_2x_5 + a_3x_1x_4 - a_4x_3x_5 \geq 0$$

$$20 \geq a_5 + a_6x_2x_5 + a_7x_1x_2 + a_8x_3^2 - 90 \geq 0$$

$$5 \geq a_9 + a_{10}x_3x_5 + a_{11}x_1x_3 + a_{12}x_3x_4 - 20 \geq 0$$

$$78 \leq x_1 \leq 102$$

$$33 \leq x_2 \leq 45$$

$$27 \leq x_i \leq 45 \quad , \quad i=3,4,5$$

a_i : cf. Appendix A

START: $x_0 = (78 , 33 , 27 , 27 , 27)$ (not feasible)

$f(x_0) = -32217$

SOLUTION:

$x^* = (78,33,29.99526,45,36.77581)$

$f(x^*) = -30665.53867$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 5$

$I(x^*) = (3 , 4 , 7 , 8 , 15)$

$u^*_{max}/u^*_{min} = 809.4/26.64 = 30.4$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	84
CLASSIFICATION:	QQR-P1-5
SOURCE:	Betts [8], Box [11,12], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 6$, $m-m_1 = 0$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = -a_1 - a_2x_1 - a_3x_1x_2 - a_4x_1x_3 - a_5x_1x_4 - a_6x_1x_5$$

CONSTRAINTS:

$$294000 \geq a_7x_1 + a_8x_1x_2 + a_9x_1x_3 + a_{10}x_1x_4 + a_{11}x_1x_5 \geq 0$$

$$294000 \geq a_{12}x_1 + a_{13}x_1x_2 + a_{14}x_1x_3 + a_{15}x_1x_4 + a_{16}x_1x_5 \geq 0$$

$$277200 \geq a_{17}x_1 + a_{18}x_1x_2 + a_{19}x_1x_3 + a_{20}x_1x_4 + a_{21}x_1x_5 \geq 0$$

$$0 \leq x_1 \leq 1000$$

$$1.2 \leq x_2 \leq 2.4$$

$$20 \leq x_3 \leq 60$$

$$9 \leq x_4 \leq 9.3$$

$$6.5 \leq x_5 \leq 7$$

a_i : cf. Appendix A

START: $x_0 = (2.52 , 2 , 37.5 , 9.25 , 6.8)$

$f(x_0) = -2351243.5$ (feasible)

SOLUTION:

$x^* = (4.53743097 , 2.4 , 60 , 9.3 , 7)$

$f(x^*) = -5280335.133$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 5$

$I(x^*) = (6 , 13 , 14 , 15 , 16)$

$u^*_{max}/u^*_{min} = .7168E6/.1914E2 = .37E5$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	85
CLASSIFICATION:	GGI-P1-2
SOURCE:	Barness [2], Caroll [17], Himmelblau [29]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 38(3)$, $m-m_1 = 0$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = -5.843E-7y_{17}(x) + 1.17E-4y_{14}(x) + 2.358E-5y_{13}(x)$$
$$+ 1.502E-6y_{16}(x) + .0321y_{12}(x) + .00423y_5(x)$$
$$+ 1.E-4c_{15}(x)/c_{16}(x) + 37.48y_2(x)/c_{12}(x) - .1365$$

CONSTRAINTS:

$$1.5x_2 - x_3 \geq 0 \qquad 704.4148 \leq x_1 \leq 906.3855$$
$$y_1(x) - 213.1 \geq 0 \qquad 68.6 \leq x_2 \leq 288.88$$
$$405.23 - y_1(x) \geq 0 \qquad 0 \leq x_3 \leq 134.75$$
$$y_{j-2}(x) - a_{j-2} \geq 0 \ , \ j=4,\dots,19 \qquad 193 \leq x_4 \leq 287.0966$$
$$b_{j-18} - y_{j-18}(x) \geq 0 \ , \ j=20,\dots,35 \qquad 25 \leq x_5 \leq 84.1988$$
$$y_4(x) - .28/.72y_5(x) \geq 0$$
$$21 - 3496y_2(x)/c_{12}(x) \geq 0$$
$$62212/c_{17}(x) - 110.6 - y_1(x) \geq 0$$

$y_j(x)$, $c_j(x)$, a_j , b_j : cf. Appendix A

START: $x_0 = (900 , 80 , 115 , 267 , 27)$

$f(x_0) = -.939$ (feasible)

SOLUTION: $x^* = (705.1803, 68.60005, 102.90001, 282.324999, 37.5850413)$

$f(x^*) = -1.90513375$

$r(x^*) = 0$

$e(x^*) = 0$

$\mu = 0$

$I(x^*) = -$

$u^*_{max}/u^*_{min} = -$

$\lambda^*_{max}/\lambda^*_{min} = .56E-3/.46E-6 = .12E4$

PROBLEM:	86 (Colville No.1)
CLASSIFICATION:	PLR-P1-1
SOURCE:	Colville [20], Himmelblau [29], Murthagh, Sargent [47]
NUMBER OF VARIABLES:	$n = 5$
NUMBER OF CONSTRAINTS:	$m_1 = 10(10)$, $m-m_1 = 0$, $b = 5$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{j=1}^{5} e_j x_j + \sum_{i=1}^{5} \sum_{j=1}^{5} c_{ij} x_i x_j + \sum_{j=1}^{5} d_j x_j^3$$

CONSTRAINTS:

$$\sum_{j=1}^{5} a_{ij} x_j - b_i \geq 0 \quad , \quad i=1,\ldots,10$$

$$0 \leq x_i \quad , \quad i=1,\ldots,5$$

$a_{ij}, b_i, c_{ij}, d_j, e_j$: cf. Appendix A

START:	x_0	=	$(0 , 0 , 0 , 0 , 1)$ (feasible)
	$f(x_0)$	=	20
SOLUTION:	x^*	=	$(.3, .33346761, .4, .42831010, .22396487)$
	$f(x^*)$	=	-32.34867897
	$r(x^*)$	=	$.70E-9$
	$e(x^*)$	=	$.94E-8$
	μ	=	4
	$I(x^*)$	=	$(3 , 5 , 6 , 9)$
	u^*_{max}/u^*_{min}	=	$11.84/.1039 = 113.9$
	$\lambda^*_{max}/\lambda^*_{min}$	=	$68.1/68.1 = 1$

PROBLEM:	87 (Colville No.6)
CLASSIFICATION:	GGI-P1-3
SOURCE:	Colville [20], Himmelblau [29]
NUMBER OF VARIABLES:	n = 6
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m - m_1 = 4$, $b = 12$

OBJECTIVE FUNCTION:

$$f(x) = f_1(x) + f_2(x)$$

$$f_1(x) = \begin{cases} 30x_1 , & 0 \le x_1 < 300 \\ 31x_1 , & 300 \le x_1 \le 400 \end{cases} \qquad f_2(x) = \begin{cases} 28x_2 , & 0 \le x_2 < 100 \\ 29x_2 , & 100 \le x_2 < 200 \\ 30x_2 , & 200 \le x_2 \le 1000 \end{cases}$$

CONSTRAINTS:

$$300 - x_1 - \frac{1}{a} x_3 x_4 \cos(b - x_6) + \frac{c}{a} d x_3^2 = 0 \qquad (a = 131.078)$$

$$- x_2 - \frac{1}{a} x_3 x_4 \cos(b + x_6) + \frac{c}{a} d x_4^2 = 0 \qquad (b = 1.48577)$$

$$- x_5 - \frac{1}{a} x_3 x_4 \sin(b + x_6) + \frac{c}{a} e x_4^2 = 0 \qquad (c = .90798)$$

$$200 - \frac{1}{a} x_3 x_4 \sin(b - x_6) + \frac{c}{a} e x_3^2 = 0 \qquad (d = \cos 1.47588) \quad (e = \sin 1.47588)$$

$$0 \le x_1 \le 400 \qquad 340 \le x_3 \le 420 \qquad -1000 \le x_5 \le 10000$$

$$0 \le x_2 \le 1000 \qquad 340 \le x_4 \le 420 \qquad 0 \le x_6 \le .5236$$

START: x_0 = (390,1000,419.5,340.5,198.175,.5)

$f(x_0)$ = 42090 (not feasible)

SOLUTION: x^* = (107.8119,196.3186,373.8307,420,
213.0713,.1532920)

$f(x^*)$ = 8927.5977

$r(x^*)$ = .10E-6

$e(x^*)$ = .74E-6

μ = 1

$I(x^*)$ = (10)

u^*_{max}/u^*_{min} = 30/.23E-6 = .13E9

$\lambda^*_{max}/\lambda^*_{min}$ = .017/.017 = 1

PROBLEM:	88 - 92 (time-optimal heat conduction)
CLASSIFICATION:	QGR-P1-(1,...,5)
SOURCE:	Schittkowski [54]
NUMBER OF VARIABLES:	$n = 2,\ldots,6$
NUMBER OF CONSTRAINTS:	$m_1 = 1$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{i=1}^{n} x_i^2$$

CONSTRAINTS:

$$\epsilon^2 - h(x) \geq 0$$

$$\epsilon = .01$$

$$h(x) = \int_0^1 \left(\sum_{i=1}^{30} \alpha_j(s)\rho_j(x) - k_o(s)\right)^2 ds$$

$$\alpha_j(s) = \mu_j^2 A_j \cos(\mu_j s)$$

$$\rho_j(x) = -\mu_j^{-2}\left(\exp(-\mu_j^2 \sum_{i=1}^{n} x_i^2) - 2\exp(-\mu_j^2 \sum_{i=2}^{n} x_i^2) + \ldots \right.$$

$$\left. + (-1)^{n-1} 2\exp(-\mu_j^2 x_n^2) + (-1)^n\right)$$

$$k_o(s) = .5(1 - s^2)$$

$$A_j = 2\sin\mu_j/(\mu_j + \sin\mu_j\cos\mu_j) \ , \quad \mu_j : \ \mu \tan\mu = 1$$

START:	x_o	=	$(.5\ ,\ -.5\ ,\ \ldots\ ,\ (-1)^{n+1}.5)$
	$f(x_o)$	=	.25n (not feasible)
SOLUTION:	x^*	=	(cf. Appendix A)
	$f(x^*)$	=	1.36265681
	$r(x^*)$	$\leq$	.30E-10 (cf. Appendix A)
	$e(x^*)$	$\leq$	.16E-2 (cf. Appendix A)
	μ	=	1
	$I(x^*)$	=	(1)
	u^*_{max}/u^*_{min}	=	1059.8/1059.8 = 1
	$\lambda^*_{max}/\lambda^*_{min}$	$\leq$	.11E9 (cf. Appendix A)

PROBLEM:	93 (transformer design)
CLASSIFICATION:	PPR-P1-6
SOURCE:	Bartholomew-Biggs [4]
NUMBER OF VARIABLES:	$n = 6$
NUMBER OF CONSTRAINTS:	$m_1 = 2$, $m-m_1 = 0$, $b = 6$

OBJECTIVE FUNCTION:

$$f(x) = .0204x_1x_4(x_1 + x_2 + x_3) + .0187x_2x_3(x_1 + 1.57x_2 + x_4)$$
$$+ .0607x_1x_4x_5^2(x_1 + x_2 + x_3)$$
$$+ .0437x_2x_3x_6^2(x_1 + 1.57x_2 + x_4)$$

CONSTRAINTS:

$$.001x_1x_2x_3x_4x_5x_6 - 2.07 \geq 0$$

$$1 - .00062x_1x_4x_5^2(x_1 + x_2 + x_3)$$
$$- .00058x_2x_3x_6^2(x_1 + 1.57x_2 + x_4) \geq 0$$

$$0 \leq x_i \quad , \quad i=1,...,6$$

START: $x_o = (5.54, 4.4, 12.02, 11.82, .702, .852)$

$f(x_o) = 137.066$ (feasible)

SOLUTION: $x^* = (5.332666, 4.656744, 10.43299, 12.08230, .7526074, .87865084)$

$f(x^*) = 135.075961$

$r(x^*) = .77E-7$

$e(x^*) = .77E-6$

$\mu = 2$

$I(x^*) = (1 \;,\; 2)$

$u^*_{max}/u^*_{min} = 71.46/62.15 = 1.15$

$\lambda^*_{max}/\lambda^*_{min} = 118.9/.21 = 562.9$

PROBLEM:	95 - 98
CLASSIFICATION:	LQR-P1-(1,...,4)
SOURCE:	Himmelblau [29], Holzman [32]
NUMBER OF VARIABLES:	$n = 6$
NUMBER OF CONSTRAINTS:	$m_1 = 4$, $m-m_1 = 0$, $b = 12$

OBJECTIVE FUNCTION:

$$f(x) = 4.3x_1 + 31.8x_2 + 63.3x_3 + 15.8x_4 + 68.5x_5 + 4.7x_6$$

CONSTRAINTS:

$$17.1x_1 + 38.2x_2 + 204.2x_3 + 212.3x_4 + 623.4x_5 + 1495.5x_6 - 169x_1x_3 - 3580x_3x_5 - 3810x_4x_5 - 18500x_4x_6 - 24300x_5x_6 \geq b_1$$

$$17.9x_1 + 36.8x_2 + 113.9x_3 + 169.7x_4 + 337.8x_5 + 1385.2x_6 - 139x_1x_3 - 2450x_4x_5 - 16600x_4x_6 - 17200x_5x_6 \geq b_2$$

$$-273x_2 - 70x_4 - 819x_5 + 26000x_4x_5 \geq b_3$$

$$159.9x_1 - 311x_2 + 587x_4 + 391x_5 + 2198x_6 - 14000x_1x_6 \geq b_4$$

$$0 \leq x_1 \leq .31 \;,\; 0 \leq x_3 \leq .068 \;,\; 0 \leq x_5 \leq .028$$

$$0 \leq x_2 \leq .046 \;,\; 0 \leq x_4 \leq .042 \;,\; 0 \leq x_6 \leq .0134$$

4 different data vectors b : cf. Appendix A

START: $x_0 = (0 , 0 , 0 , 0 , 0 , 0)$ (not feasible)

$f(x_0) = 0$

SOLUTION:

$x^* = (0,0,0,0,0,.0033233033)$ (95,96)

$= (.2685649,0,0,0,.028,.0134)$ (97,98)

$f(x^*) = .015619514$ (95,96) 3.1358091 (97,98)

$r(x^*) = .21E-9$ (95,96) 0

$e(x^*) = 0$

$\mu = 6$

$I(x^*) = (1,5,6,7,8,9)$ (95,96) (1,6,7,8,15,16)

$u^*_{max}/u^*_{min} = 66.8/.003=.2E5$ (95,96) 200/.251=.8E3

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	99
CLASSIFICATION:	GGR-P1-3
SOURCE:	Betts [8]
NUMBER OF VARIABLES:	$n = 7$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 2$, $b = 14$

OBJECTIVE FUNCTION:

$$f(x) = -r_8(x)^2$$

$$r_1(x) = 0 , \quad r_i(x) = a_i(t_i - t_{i-1})\cos x_{i-1} + r_{i-1}(x), \ i=2,..,8$$

CONSTRAINTS:

$$q_8(x) - 1.E5 = 0$$

$$s_8(x) - 1.E3 = 0$$

$$0 \le x_i \le 1.58 , \ i=1,...,7$$

$$q_1(x) = s_1(x) = 0$$

$$q_i(x) = .5(t_i - t_{i-1})^2(a_i \sin x_{i-1} - b) + (t_i - t_{i-1})s_{i-1}(x) + q_{i-1}(x)$$

$$s_i(x) = (t_i - t_{i-1})(a_i \sin x_{i-1} - b) + s_{i-1}(x) , \quad i=2,...,8$$

a_i, t_i, b : cf. Appendix A

START:

x_o = (.5 , .5 , .5 , .5 , .5 , .5 , .5)

$f(x_o)$ = -.7763605E9 (not feasible)

SOLUTION:

x^* = (.5424603,.5290159,.5084506,.4802693, .4512352,.4091878,.3527847)

$f(x^*)$ = -.831079892E9

$r(x^*)$ = .30E-7

$e(x^*)$ = .31E4 , $\|\nabla f(x^*)\|$ = .32E9

μ = 0

$I(x^*)$ = -

u^*_{max}/u^*_{min} = .1934E5/.4194E2 = .46E3

$\lambda^*_{max}/\lambda^*_{min}$ = .50E9/.84E8 = 5.95

PROBLEM:	100
CLASSIFICATION:	PPR-P1-7
SOURCE:	Asaadi [1], Charalambous [18], Wong [59]
NUMBER OF VARIABLES:	$n = 7$
NUMBER OF CONSTRAINTS:	$m_1 = 4$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + 3(x_4 - 11)^2 + 10x_5^6 + 7x_6^2 + x_7^4 - 4x_6x_7 - 10x_6 - 8x_7$$

CONSTRAINTS:

$$127 - 2x_1^2 - 3x_2^4 - x_3 - 4x_4^2 - 5x_5 \geq 0$$

$$282 - 7x_1 - 3x_2 - 10x_3^2 - x_4 + x_5 \geq 0$$

$$196 - 23x_1 - x_2^2 - 6x_6^2 + 8x_7 \geq 0$$

$$-4x_1^2 - x_2^2 + 3x_1x_2 - 2x_3^2 - 5x_6 + 11x_7 \geq 0$$

START: $x_0 = (1 , 2 , 0 , 4 , 0 , 1 , 1)$

$f(x_0) = 714$ (feasible)

SOLUTION: $x^* = (2.330499, 1.951372, -.4775414, 4.365726, -.6244870, 1.038131, 1.594227)$

$f(x^*) = 680.6300573$

$r(x^*) = .90E-7$

$e(x^*) = .36E-8$

$\mu = 2$

$I(x^*) = (1 , 4)$

$u^*_{max}/u^*_{min} = 1.140/.3686 = 3.09$

$\lambda^*_{max}/\lambda^*_{min} = 46.6/4.34 = 10.7$

PROBLEM:	101 - 103
CLASSIFICATION:	PPR-P1-(8,9,10)
SOURCE:	Beck, Ecker [5], Dembo [22]
NUMBER OF VARIABLES:	n = 7
NUMBER OF CONSTRAINTS:	$m_1 = 6$, $m-m_1 = 0$, b = 14

OBJECTIVE FUNCTION: 101 : a=-.25 , 102 : a=.125 , 103 : a=.5

$$f(x) = 10x_1x_2^{-1}x_4^{2}x_6^{-3}x_7^{a} + 15x_1^{-1}x_2^{-2}x_3x_4x_5^{-1}x_7^{-.5} + 20x_1^{-2}x_2x_4^{-1}x_5^{-2}x_6 + 25x_1^{2}x_2^{2}x_3^{-1}x_5^{.5}x_6^{-2}x_7$$

CONSTRAINTS:

$$1 - .5x_1^{.5}x_3^{-1}x_6^{-2}x_7 - .7x_1^{3}x_2x_3^{-2}x_6x_7^{.5} - .2x_2^{-1}x_3x_4^{-.5}x_6^{2/3}x_7^{1/4} \geq 0$$

$$1 - 1.3x_1^{-.5}x_2x_3^{-1}x_5^{-1}x_6 - .8x_3x_4^{-1}x_5^{-1}x_6^{2} - 3.1x_1^{-1}x_2^{.5}x_4^{-2}x_5^{-1}x_6^{1/3} \geq 0$$

$$1 - 2x_1x_3^{-1.5}x_5x_6^{-1}x_7^{1/3} - .1x_2x_3^{-.5}x_5x_6^{-1}x_7^{-.5} - x_1^{-1}x_2x_3^{.5}x_5 - .65x_2^{-2}x_3x_5x_6^{-1}x_7 \geq 0$$

$$1 - .2x_1^{-2}x_2x_4^{-1}x_5^{.5}x_7^{1/3} - .3x_1^{.5}x_2^{2}x_3x_4^{1/3}x_7^{1/4}x_5^{-2/3} - .4x_1^{-3}x_2^{-2}x_3x_5x_7^{3/4} - .5x_3^{-2}x_4x_7^{.5} \geq 0$$

$100 \leq f(x) \leq 3000$, $.1 \leq x_i \leq 10$, i=1,..,6, $.01 \leq x_7 \leq 10$

START: x_0 = (6 , 6 , 6 , 6 , 6 , 6 , 6) (not feas.)

$f(x_0)$ = 2205.868 , 2206.889 , 2208.886

SOLUTION: x^* = (cf. Appendix A)

		101	102	103
$f(x^*)$	=	1809.76476	911.880571	543.667958
$r(x^*)$	=	.10E-10	.27E-10	.61E-11
$e(x^*)$	=	.59E-6	.14E-6	.92E-7
μ	=	3	3	4
$I(x^*)$	=	(2,3,13)	(1,2,3)	(1,2,3,4)
u^*_{max}/u^*_{min}	=	4567/.81E-6	2173/21.13	1287/38.71
$\lambda^*_{max}/\lambda^*_{min}$	=	.43E4/.51E2=85	.23E4/.32E2=73	.46E3/.88E2=5.2

PROBLEM:	104 (optimal reactor design)
CLASSIFICATION:	PPR-P1-11
SOURCE:	Dembo [22], Rijckaert [53]
NUMBER OF VARIABLES:	$n = 8$
NUMBER OF CONSTRAINTS:	$m_1 = 6$, $m-m_1 = 0$, $b = 16$

OBJECTIVE FUNCTION:

$$f(x) = .4x_1^{.67}x_7^{-.67} + .4x_2^{.67}x_8^{-.67} + 10 - x_1 - x_2$$

CONSTRAINTS:

$$1 - .0588x_5x_7 - .1x_1 \geq 0$$

$$1 - .0588x_6x_8 - .1x_1 - .1x_2 \geq 0$$

$$1 - 4x_3x_5^{-1} - 2x_3^{-.71}x_5^{-1} - .0588x_3^{-1.3}x_7 \geq 0$$

$$1 - 4x_4x_6^{-1} - 2x_4^{-.71}x_6^{-1} - .0588x_4^{-1.3}x_8 \geq 0$$

$$1 \leq f(x) \leq 4.2$$

$$.1 \leq x_i \leq 10 \quad , \quad i=1,\dots,8$$

START: $x_o = (6,3,.4,.2,6,6,1,.5)$ (not feasible)

$f(x_o) = 3.65$

SOLUTION:

$x^* = (6.465114,2.232709,.6673975,.5957564,$
$5.932676,5.527235,1.013322,.4006682)$

$f(x^*) = 3.9511634396$

$r(x^*) = .58E-10$

$e(x^*) = .31E-10$

$\mu = 4$

$I(x^*) = (1 , 2 , 3 , 4)$

$u^*_{max}/u^*_{min} = 6.206/.8472 = 7.32$

$\lambda^*_{max}/\lambda^*_{min} = 1.87/.043 = 43.2$

PROBLEM:	105 (maximum-likelihood estimation)
CLASSIFICATION:	GLR-P1-2
SOURCE:	Bracken, McCormick [13]
NUMBER OF VARIABLES:	n = 8
NUMBER OF CONSTRAINTS:	$m_1 = 1(1)$, $m-m_1 = 0$, $b = 16$

OBJECTIVE FUNCTION:

$$f(x) = -\sum_{i=1}^{235} \ln((a_i(x) + b_i(x) + c_i(x))/\sqrt{2\pi})$$

$$a_i(x) = x_1/x_6 \exp(-(y_i - x_3)^2/(2x_6^2))$$

$$b_i(x) = x_2/x_7 \exp(-(y_i - x_4)^2/(2x_7^2)) \quad , \quad i=1,\ldots,235$$

$$c_i(x) = (1 - x_2 - x_1)/x_8 \exp(-(y_i - x_5)^2/(2x_8^2))$$

y_i : cf. Appendix A

CONSTRAINTS:

$1 - x_1 - x_2 \geq 0$

$.001 \leq x_i \leq .499$, $i=1,2$ $\qquad 100 \leq x_3 \leq 180$

$130 \leq x_4 \leq 210 \qquad 170 \leq x_5 \leq 240 \qquad 5 \leq x_i \leq 25$, $i=6,7,8$

START: x_o = (.1,.2,100,125,175,11.2,13.2,15.8)

$f(x_o)$ = 1297.6693 (feasible)

SOLUTION: x^* = (.4128928,.4033526,131.2613,164.3135, 217.4222,12.28018,15.77170,20.74682)

$f(x^*)$ = 1138.416240

$r(x^*)$ = 0

$e(x^*)$ = 0

μ = 0

$I(x^*)$ = -

u^*_{max}/u^*_{min} = -

$\lambda^*_{max}/\lambda^*_{min}$ = .26E4/.28E-2 = .92E6

PROBLEM:	106 (heat exchanger design)
CLASSIFICATION:	LQR-P1-5
SOURCE:	Avriel, Williams [2], Dembo [22]
NUMBER OF VARIABLES:	$n = 8$
NUMBER OF CONSTRAINTS:	$m_1 = 6(3)$, $m-m_1 = 0$, $b = 16$

OBJECTIVE FUNCTION:

$$f(x) = x_1 + x_2 + x_3$$

CONSTRAINTS:

$$1 - .0025(x_4 + x_6) \geq 0$$

$$1 - .0025(x_5 + x_7 - x_4) \geq 0$$

$$1 - .01(x_8 - x_5) \geq 0$$

$$x_1 x_6 - 833.33252 x_4 - 100 x_1 + 83333.333 \geq 0$$

$$x_2 x_7 - 1250 x_5 - x_2 x_4 + 1250 x_4 \geq 0$$

$$x_3 x_8 - 1250000 - x_3 x_5 + 2500 x_5 \geq 0$$

$$100 \leq x_1 \leq 10000$$

$$1000 \leq x_i \leq 10000 \quad , \quad i=2,3$$

$$10 \leq x_i \leq 1000 \quad , \quad i=4,\ldots,8$$

START: $x_o = (5000,5000,5000,200,350,150,225,425)$

$f(x_o) = 15000$ (not feasible)

SOLUTION: $x^* = (579.3167,1359.943,5110.071,182,0174,$
$295.5985,217.9799,286.4162,395,5979)$

$f(x^*) = 7049.330923$

$r(x^*) = 0$

$e(x^*) = .19E-4$

$\mu = 6$

$I(x^*) = (1 , 2 , 3 , 4 , 5 , 6)$

$u^*_{max}/u^*_{min} = 5210/.00848 = .61E6$

$\lambda^*_{max}/\lambda^*_{min} = .81E-3/.38E-3 = 2.11$

PROBLEM:	107 (static power scheduling)
CLASSIFICATION:	PGR-P1-4
SOURCE:	Bartholomew-Biggs [4]
NUMBER OF VARIABLES:	$n = 9$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 6$, $b = 8$

OBJECTIVE FUNCTION:

$$f(x) = 3000x_1 + 1000x_1^3 + 2000x_2 + 666.667x_2^3$$

CONSTRAINTS:

$$.4 - x_1 + 2cx_5^2 - x_5x_6(dy_1 + cy_2) - x_5x_7(dy_3 + cy_4) = 0$$

$$.4 - x_2 + 2cx_6^2 + x_5x_6(dy_1 - cy_2) + x_6x_7(dy_5 - cy_6) = 0$$

$$.8 + 2cx_7^2 + x_5x_7(dy_3 - cy_4) - x_6x_7(dy_5 + cy_6) = 0$$

$$.2 - x_3 + 2dx_5^2 + x_5x_6(cy_1 - dy_2) + x_5x_7(cy_3 - dy_4) = 0$$

$$.2 - x_4 + 2dx_6^2 - x_5x_6(cy_1 + dy_2) - x_6x_7(cy_5 + dy_6) = 0$$

$$-.337 + 2dx_7^2 - x_5x_7(cy_3 + dy_4) + x_6x_7(cy_5 - dy_6) = 0$$

$$0 \le x_i \ , \ i=1,2 \quad , \quad .90909 \le x_i \le 1.0909 \ , \ i=5,6,7$$

$$y_1 = \sin x_8 \ , \quad y_2 = \cos x_8 \ , \quad y_3 = \sin x_9$$

$$y_4 = \cos x_9 \ , \quad y_5 = \sin(x_8-x_9) \ , \quad y_6 = \cos(x_8-x_9)$$

$$c = (48.4/50.176)\sin .25 \ , \quad d = (48.4/50.176)\cos .25$$

START:

x_0	=	(.8,.8,.2,.2,1.0454,1.0454,0,0)
$f(x_0)$	=	4853.3335 (not feasible)

SOLUTION:

x^*	=	(.6670095,1.022388,.2282879,.1848217, 1.090900,1.090900,1.069036,.1066126, -.3387867)
$f(x^*)$	=	5055.011803
$r(x^*)$	=	.18E-9
$e(x^*)$	=	0
μ	=	3
$I(x^*)$	=	(6 , 7 , 8)
u^*_{max}/u^*_{min}	=	5208/0
$\lambda^*_{max}/\lambda^*_{min}$	=	-

PROBLEM:	108
CLASSIFICATION:	QQR-P1-6
SOURCE:	Himmelblau [29], Pearson [49]
NUMBER OF VARIABLES:	$n = 9$
NUMBER OF CONSTRAINTS:	$m_1 = 13$, $m-m_1 = 0$, $b = 1$

OBJECTIVE FUNCTION:

$$f(x) = -.5(x_1x_4 - x_2x_3 + x_3x_9 - x_5x_9 + x_5x_8 - x_6x_7)$$

CONSTRAINTS:

$$1 - x_3^2 - x_4^2 \geq 0 \qquad 1 - x_9^2 \geq 0$$

$$1 - x_5^2 - x_6^2 \geq 0 \qquad 1 - x_1^2 - (x_2 - x_9)^2 \geq 0$$

$$1 - (x_1 - x_5)^2 - (x_2 - x_6)^2 \geq 0$$

$$1 - (x_1 - x_7)^2 - (x_2 - x_8)^2 \geq 0$$

$$1 - (x_3 - x_5)^2 - (x_4 - x_6)^2 \geq 0$$

$$1 - (x_3 - x_7)^2 - (x_4 - x_8)^2 \geq 0$$

$$1 - x_7^2 - (x_8 - x_9)^2 \geq 0 \qquad x_1x_4 - x_2x_3 \geq 0$$

$$x_3x_9 \geq 0 \qquad -x_5x_9 \geq 0$$

$$x_5x_8 - x_6x_7 \geq 0 \qquad 0 \leq x_9$$

START: x_0 = (1,1,1,1,1,1,1,1,1) (not feasible)

$f(x_0) = 0$

SOLUTION: x^* = (.8841292,.4672425,.03742076,.9992996, .8841292,.4672424,.03742076,.9992996, .26E-19)

$f(x^*) = -.8660254038$

$r(x^*) = .39E-9$ $\qquad e(x^*) = .33E-11$

$u = 9$ $\qquad I(x^*) = (1,3,4,6,7,9,11,12,14)$

$u^*_{max}/u^*_{min} = .1443/0$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	109
CLASSIFICATION:	PGR-P1-5
SOURCE:	Beuneu [9]
NUMBER OF VARIABLES:	$n = 9$
NUMBER OF CONSTRAINTS:	$m_1 = 4(2)$, $m-m_1 = 6$, $b = 16$

OBJECTIVE FUNCTION:

$$f(x) = 3x_1 + 1.E\text{-}6x_1^3 + 2x_2 + .522074E\text{-}6x_2^3$$

CONSTRAINTS:

$$x_4 - x_3 + .55 \geq 0 \qquad x_3 - x_4 + .55 \geq 0$$

$$2250000 - x_1^2 - x_8^2 \geq 0 \qquad 2250000 - x_2^2 - x_9^2 \geq 0$$

$$x_5x_6\sin(-x_3 - \tfrac{1}{4}) + x_5x_7\sin(-x_4 - \tfrac{1}{4}) + 2bx_5^2 - ax_1 + 400a = 0$$

$$x_5x_6\sin(x_3 - \tfrac{1}{4}) + x_6x_7\sin(x_3 - x_4 - \tfrac{1}{4}) + 2bx_6^2 - ax_2 + 400a = 0$$

$$x_5x_7\sin(x_4 - \tfrac{1}{4}) + x_6x_7\sin(x_4 - x_3 - \tfrac{1}{4}) + 2bx_7^2 + 881.779a = 0$$

$$ax_8 + x_5x_6\cos(-x_3 - \tfrac{1}{4}) + x_5x_7\cos(-x_4 - \tfrac{1}{4}) - 200a - 2cx_5^2 + .7533E\text{-}3ax_5^2 = 0$$

$$ax_9 + x_5x_6\cos(x_3 - \tfrac{1}{4}) + x_6x_7\cos(x_3 - x_4 - \tfrac{1}{4}) - 2cx_6^2 + .7533E\text{-}3ax_6^2 - 200a = 0$$

$$x_5x_7\cos(x_4 - \tfrac{1}{4}) + x_6x_7\cos(x_4 - x_3 - \tfrac{1}{4}) - 2cx_7^2 + 22.938a + .7533E\text{-}3ax_7^2 = 0$$

$0 \leq x_i$, $i=1,2$ $\qquad$ $-.55 \leq x_i \leq .55$, $i=3,4$

$196 \leq x_i \leq 252$, $i=5,6,7$ $\qquad$ $-400 \leq x_i \leq 800$, $i=8,9$

$a = 50.176$, $b = \sin .25$, $c = \cos .25$

START: $x_0 = (0,\ldots,0)$ $\qquad$ $f(x_0) = 0$ (not feasible)

SOLUTION: $\qquad$ $f(x^*) = 5362.06928$

x^*	= (cf. Appendix A)		
$r(x^*)$	= .36E-7	$e(x^*)$	= 0
μ	= 3	$I(x^*)$	= (1 , 16 , 17)
u^*_{max}/u^*_{min}	= 12.53/.13E-10 = .95E12		
$\lambda^*_{max}/\lambda^*_{min}$	= -		

PROBLEM:	110
CLASSIFICATION:	GBR-P1-1
SOURCE:	Himmelblau [29], Paviani [48]
NUMBER OF VARIABLES:	n = 10
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 0$, $b = 20$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{i=1}^{10} [(\ln(x_i - 2))^2 + (\ln(10 - x_i))^2 - (\prod_{i=1}^{10} x_i)^{.2}$$

CONSTRAINTS:

$$2.001 \le x_i \le 9.999 \quad , \quad i=1,\ldots,10$$

START:	x_o	=	$(9,\ldots,9)$ (feasible)
	$f(x_o)$	=	-43.134337
SOLUTION:	x^*	=	$(9.35025655,\ldots,9.35025655)$
	$f(x^*)$	=	-45.77846971
	$r(x^*)$	=	0
	$e(x^*)$	=	0
	μ	=	0
	$I(x^*)$	=	-
	u^*_{max}/u^*_{min}	=	-
	$\lambda^*_{max}/\lambda^*_{min}$	=	$6.92/6.52 = 1.06$

PROBLEM:	111
CLASSIFICATION:	GGR-P1-4
SOURCE:	Bracken, McCormick [13], Himmelblau [29], White [58]
NUMBER OF VARIABLES:	$n = 10$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3$, $b = 20$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{j=1}^{10} \exp(x_j)(c_j + x_j - \ln(\sum_{k=1}^{10} \exp(x_k)))$$

c_j : cf. Appendix A

CONSTRAINTS:

$$\exp(x_1) + 2\exp(x_2) + 2\exp(x_3) + \exp(x_6) + \exp(x_{10}) - 2 = 0$$

$$\exp(x_4) + 2\exp(x_5) + \exp(x_6) + \exp(x_7) - 1 = 0$$

$$\exp(x_3) + \exp(x_7) + \exp(x_8) + 2\exp(x_9) + \exp(x_{10}) - 1 = 0$$

$$-100 \le x_i \le 100 \quad , \quad i=1,\dots,10$$

START: $x_o = (-2.3 , \dots ,-2.3)$ (not feasible)

$f(x_o) = -21.015$

SOLUTION: $x^* = (-3.201212,-1.912060,-.2444413,-6.537489,$
$-.7231524,-7.267738,-3.596711,-4.017769,$
$-3.287462,-2.335582)$

$f(x^*) = -47.76109026$

$r(x^*) = .34E-9$ $\qquad$ $e(x^*) = .14E-3$

$u = 0$ $\qquad$ $I(x^*) = -$

$u^*_{max}/u^*_{min} = 15.22/9.785 = 1.56$

$\lambda^*_{max}/\lambda^*_{min} = .11/.70E-3 = 160.1$

PROBLEM:	112 (chemical equilibrium)
CLASSIFICATION:	GLR-P1-3
SOURCE:	Bracken, McCormick [13], Himmelblau [29], White [58]
NUMBER OF VARIABLES:	$n = 10$
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 3(3)$, $b = 10$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{j=1}^{10} x_j(c_j + \ln \frac{x_j}{x_1 + \ldots + x_{10}})$$

c_j : cf. Appendix A

CONSTRAINTS:

$$x_1 + 2x_2 + 2x_3 + x_6 + x_{10} - 2 = 0$$

$$x_4 + 2x_5 + x_6 + x_7 - 1 = 0$$

$$x_3 + x_7 + x_8 + 2x_9 + x_{10} = 0$$

$$1.E-6 \le x_i \quad , \quad i=1,\ldots,10$$

START: $x_o = (.1 , \ldots , .1)$ (not feasible)

$f(x_o) = -20.961$

SOLUTION: $x^* = (.01773548,.08200180,.8825646,.7233256E-3,$
$.4907851,.4335469E-3,.01727298,$
$.007765639,.01984929,.05269826)$

$f(x^*) = -47.707579$

$r(x^*) = .23E-7$ $e(x^*) = .43E-6$

$u = 2$ $I(x^*) = (4 , 6)$

$u^*_{max}/u^*_{min} = 15.02/.262E-3 = .57E5$

$\lambda^*_{max}/\lambda^*_{min} = 191/8.98 = 21.3$

PROBLEM:	113 (Wong No.2)
CLASSIFICATION:	QQR-P1-7
SOURCE:	Asaadi [1], Charalambous [18], Wong [59]
NUMBER OF VARIABLES:	$n = 10$
NUMBER OF CONSTRAINTS:	$m_1 = 8(3)$, $m-m_1 = 0$, $b = 0$

OBJECTIVE FUNCTION:

$$f(x) = x_1^2 + x_2^2 + x_1x_2 - 14x_1 - 16x_2 + (x_3 - 10)^2 + 4(x_4 - 5)^2 + (x_5 - 3)^2 + 2(x_6 - 1)^2 + 5x_7^2 + 7(x_8 - 11)^2 + 2(x_9 - 10)^2 + (x_{10} - 7)^2 + 45$$

CONSTRAINTS:

$$105 - 4x_1 - 5x_2 + 3x_7 - 9x_8 \geq 0$$
$$-10x_1 + 8x_2 + 17x_7 - 2x_8 \geq 0$$
$$8x_1 - 2x_2 - 5x_9 + 2x_{10} + 12 \geq 0$$
$$-3(x_1 - 2)^2 - 4(x_2 - 3)^2 - 2x_3^2 + 7x_4 + 120 \geq 0$$
$$-5x_1^2 - 8x_2 - (x_3 - 6)^2 + 2x_4 + 40 \geq 0$$
$$-.5(x_1 - 8)^2 - 2(x_2 - 4)^2 - 3x_5^2 + x_6 + 30 \geq 0$$
$$-x_1^2 - 2(x_2 - 2)^2 + 2x_1x_2 - 14x_5 + 6x_6 \geq 0$$
$$3x_1 - 6x_2 - 12(x_9 - 8)^2 + 7x_{10} \geq 0$$

START: $x_0 = (2,3,5,5,1,2,7,3,6,10)$ (feasible)

$f(x_0) = 753$

SOLUTION: $x^* = (2.171996, 2.363683, 8.773926, 5.095984, .9906548, 1.430574, 1.321644, 9.828726, 8.280092, 8.375927)$

$f(x^*) = 24.3062091$

$r(x^*) = .12E-8$ $e(x^*) = .46E-9$

$u = 6$ $I(x^*) = (1,2,3,4,5,7)$

$u^*_{max}/u^*_{min} = 1.717/.02055 = 83.5$

$\lambda^*_{max}/\lambda^*_{min} = 7.79/2.24 = 3.48$

PROBLEM:	114 (alkylation process)
CLASSIFICATION:	QGR-P1-6
SOURCE:	Bracken, McCormick [13]
NUMBER OF VARIABLES:	$n = 10$
NUMBER OF CONSTRAINTS:	$m_1 = 8(4)$, $m-m_1 = 3(1)$, $b = 20$

OBJECTIVE FUNCTION:

$$f(x) = 5.04x_1 + .035x_2 + 10x_3 + 3.36x_5 - .063x_4x_7$$

CONSTRAINTS:

$$g_1(x) = 35.82 - .222x_{10} - bx_9 \geq 0$$
$$g_2(x) = -133 + 3x_7 - ax_{10} \geq 0$$
$$g_3(x) = -g_1(x) + x_9(1/b - b) \geq 0$$
$$g_4(x) = -g_2(x) + (1/a - a)x_{10} \geq 0$$
$$g_5(x) = 1.12x_1 + .13167x_1x_8 - .00667x_1x_8^2 - ax_4 \geq 0$$
$$g_6(x) = 57.425 + 1.098x_8 - .038x_8^2 + .325x_6 - ax_7 \geq 0$$
$$g_7(x) = -g_5(x) + (1/a - a)x_4 \geq 0$$
$$g_8(x) = -g_6(x) + (1/a - a)x_7 \geq 0$$
$$g_9(x) = 1.22x_4 - x_1 - x_5 = 0$$
$$g_{10}(x) = 98000x_3/(x_4x_9 + 1000x_3) - x_6 = 0 \qquad a = .99$$
$$g_{11}(x) = (x_2 + x_5)/x_1 - x_8 = 0 \qquad b = .9$$

bounds: cf. Appendix A

START: $x_0 = (1745,12000,110,3048,1974,89.2,92.8,8,3.6,145)$ (not feasible)

$f(x_0) = -872.3872$

SOLUTION: $x^* = (1698.096,15818.73,54.10228,3031.226,2000,90.11537,95,10.49336,1.561636,153.53535)$

$f(x^*) = -1768.80696$

$r(x^*) = 0$ $\qquad e(x^*) = .16E-5$

$u = 6$ $\qquad I(x^*) = (2,3,5,6,23,25)$

$u^*_{max}/u^*_{min} = 311.8/.6778 = 460$

$\lambda^*_{max}/\lambda^*_{min} = .14E-4/.14E-4 = 1$

PROBLEM:	116 (3-stage membrane separation)
CLASSIFICATION:	LQR-P1-6
SOURCE:	Dembo [21,22]
NUMBER OF VARIABLES:	$n = 13$
NUMBER OF CONSTRAINTS:	$m_1 = 15(5)$, $m-m_1 = 0$, $b = 26$

OBJECTIVE FUNCTION:

$$f(x) = x_{11} + x_{12} + x_{13}$$

CONSTRAINTS:

$$x_3 - x_2 \geq 0 \qquad x_2 - x_1 \geq 0$$

$$1 - .002x_7 + .002x_8 \geq 0 \qquad 50 \leq f(x) \leq 250$$

$$x_{13} - 1.262626x_{10} + 1.231059x_3x_{10} \geq 0$$

$$x_5 - .03475x_2 - .975x_2x_5 + .00975x_2^2 \geq 0$$

$$x_6 - .03475x_3 - .975x_3x_6 + .00975x_3^2 \geq 0$$

$$x_5x_7 - x_1x_8 - x_4x_7 + x_4x_8 \geq 0$$

$$1 - .002(x_2x_9 + x_5x_8 - x_1x_8 - x_6x_9) - x_5 - x_6 \geq 0$$

$$x_2x_9 - x_3x_{10} - x_6x_9 - 500x_2 + 500x_6 + x_2x_{10} \geq 0$$

$$x_2 - .9 - .002(x_2x_{10} - x_3x_{10}) \geq 0$$

$$x_4 - .03475x_1 - .975x_1x_4 + .00975x_1^2 \geq 0$$

$$x_{11} - 1.262626x_8 + 1.231059x_1x_8 \geq 0$$

$$x_{12} - 1.262626x_9 + 1.231059x_2x_9 \geq 0$$

bounds: cf. Appendix A

START: $x_0 = (.5,.8,.9,.1,.14,.5,489,80,650,450,150,150,150)$ (not feasible)

$f(x_0) = 450$

SOLUTION: $x^* = (.8037703,.8999860,.9709724,.09999952,.1908154,.4605717,574.0803,74.08043,500.0162,.1,20.23413,77.34755,.00673039)$

$f(x^*) = 97.588409$

$r(x^*) = 0$ $\qquad e(x^*) = 0$

$u = 14$ $\qquad I(x^*) = (3,6,\ldots,15,25,28,32)$

$u^*_{max}/u^*_{min} = 2088/.423E-3 = .49E7$

$\lambda^*_{max}/\lambda^*_{min} = -$

PROBLEM:	117 (Colville No.2, Shell Dual)
CLASSIFICATION:	PQR-P1-1
SOURCE:	Colville [20], Himmelblau [29]
NUMBER OF VARIABLES:	n = 15
NUMBER OF CONSTRAINTS:	$m_1 = 5$, $m-m_1 = 0$, $b = 15$

OBJECTIVE FUNCTION:

$$f(x) = -\sum_{j=1}^{10} b_j x_j + \sum_{j=1}^{5}\sum_{k=1}^{5} c_{kj} x_{10+k} x_{10+j} + 2\sum_{j=1}^{5} d_j x_{10+j}^3$$

CONSTRAINTS:

$$2\sum_{k=1}^{5} c_{kj} x_{10+k} + 3 d_j x_{10+j}^2 + e_j - \sum_{k=1}^{10} a_{kj} x_k \ge 0 \quad , \quad j=1,\ldots,5$$

$$0 \le x_i \quad , \quad i=1,\ldots,15$$

$a_{ij}, b_j, c_{ij}, d_j, e_j$: cf. Appendix A

START: x_o = .001(1,1,1,1,1,1,60000,1,1,1,1,1,1,1,1)

$f(x_o)$ = 2400.1053 (feasible)

SOLUTION: x^* = (0,0,5.174136,0,3.061093,11.83968,0,0, .1039071,0,.2999929,.3334709,.3999910, .4283145,.2239607)

$f(x^*)$ = 32.348679

$r(x^*)$ = 0 $e(x^*)$ = .35E-4

u = 11 $I(x^*)$ = (1,...,7,9,12,13, 15)

u^*_{max}/u^*_{min} = 56.75/.2240 = 253

$\lambda^*_{max}/\lambda^*_{min}$ = 3.30/.10 = 32.3

PROBLEM:	118
CLASSIFICATION:	QLR-P1-2
SOURCE:	Bartholomew-Biggs [4]
NUMBER OF VARIABLES:	n = 15
NUMBER OF CONSTRAINTS:	$m_1 = 29(29)$, $m-m_1 = 0$, $b = 30$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{k=0}^{4} (2.3x_{3k+1} + .0001x_{3k+1}^2 + 1.7x_{3k+2} + .0001x_{3k+2}^2 + 2.2x_{3k+3} + .00015x_{3k+3}^2)$$

CONSTRAINTS:

$0 \le x_{3j+1} - x_{3j-2} + 7 \le 13$ $\quad$ $0 \le x_{3j+2} - x_{3j-1} + 7 \le 14$

$0 \le x_{3j+3} - x_{3j} + 7 \le 13$ $\quad$ $j=1,\ldots,4$

$x_1 + x_2 + x_3 - 60 \ge 0$ $\quad$ $x_4 + x_5 + x_6 - 50 \ge 0$

$x_7 + x_8 + x_9 - 70 \ge 0$ $\quad$ $x_{10} + x_{11} + x_{12} - 85 \ge 0$

$x_{13} + x_{14} + x_{15} - 100 \ge 0$

$8 \le x_1 \le 21$ $\quad$ $43 \le x_2 \le 57$ $\quad$ $3 \le x_3 \le 16$

$0 \le x_{3k+1} \le 90$ $\quad$ $0 \le x_{3k+2} \le 120$ $\quad$ $0 \le x_{3k+3} \le 60$

$k=1,\ldots,4$

START: x_0 = (20,55,15,20,60,20,20,60,20,20,60,20,20,60,20) (feasible)

$f(x_0)$ = 664.82045000

SOLUTION: x^* = (8,49,3,1,56,0,1,63,6,3,70,12,5,77,18)

$f(x^*)$ = 664.8204500

$r(x^*)$ = 0

$e(x^*)$ = 0

μ = 15

$I(x^*)$ = (1,17,18,19,20,22,23,24,25,27,28,29,30,32,35)

u^*_{max}/u^*_{min} = 2.941/.04860 = 60.5

$\lambda^*_{max}/\lambda^*_{min}$ = -

PROBLEM:	119 (Colville No.7)
CLASSIFICATION:	PLR-P1-2
SOURCE:	Colville [20], Himmelblau [29]
NUMBER OF VARIABLES:	n = 16
NUMBER OF CONSTRAINTS:	$m_1 = 0$, $m-m_1 = 8(8)$, $b = 32$

OBJECTIVE FUNCTION:

$$f(x) = \sum_{i=1}^{16} \sum_{j=1}^{16} a_{ij}(x_i^2 + x_i + 1)(x_j^2 + x_j + 1)$$

CONSTRAINTS:

$$\sum_{j=1}^{16} b_{ij}x_j - c_i = 0 \quad , \quad i=1,\ldots,8$$

$$0 \le x_i \le 5 \quad , \quad i=1,\ldots,16$$

a_{ij}, b_{ij}, c_i : cf. Appendix A

START: x_o = (10,...,10) (not feasible)

$f(x_o)$ = 566766

SOLUTION: x^* = (.03984735,.7919832,.2028703,.8443579,
1.126991,.9347387,1.681962,.1553009,
1.567870,0,0,0,.6602041,0,.6742559,0)

$f(x^*)$ = 244.899698

$r(x^*)$ = .26E-9 $e(x^*)$ = .36E-8

u = 5 $I(x^*)$ = (10,11,12,14,16)

u^*_{max}/u^*_{min} = 95.99/4.201 = 22.9

$\lambda^*_{max}/\lambda^*_{min}$ = 39.2/25.1 = 1.56

CONSTANT DATA

This appendix summarizes constant data, parts of the definition of the problem functions, and numerical results which would break the documentation scheme used in Chapter IV to describe the test problems. The corresponding abbreviations are explained in the test problem documentations.

No. 57:

i	a_i	b_i	i	a_i	b_i
1	8	.49	23	22	.41
2	8	.49	24	22	.40
3	10	.48	25	24	.42
4	10	.47	26	24	.40
5	10	.48	27	24	.40
6	10	.47	28	26	.41
7	12	.46	29	26	.40
8	12	.46	30	26	.41
9	12	.45	31	28	.41
10	12	.43	32	28	.40
11	14	.45	33	30	.40
12	14	.43	34	30	.40
13	14	.43	35	30	.38
14	16	.44	36	32	.41
15	16	.43	37	32	.40
16	16	.43	38	34	.40
17	18	.46	39	36	.41
18	18	.45	40	36	.38
19	20	.42	41	38	.40
20	20	.42	42	38	.40
21	20	.43	43	40	.39
22	22	.41	44	42	.39

Table 5: Data for test problem no. 57.

No. 67:

Let $y_i = y_i(x)$. The functions are described by a subprogram:

```
   y2 = 1.6x1
10 y3 = 1.22y2 - x1
   y6 = (x2 + y3)/x1
   y2c = .01x1(112 + 13.167y6 - .6667y6^2)
   if |y2c - y2| ≤ .001  goto 30 else goto 20
20 y2 = y2c
   goto 10
30 y4 = 93
40 y5 = 86.35 + 1.098y6 - .038y6^2 + .325(y4 - 89)
   y8 = 3y5 - 133
   y7 = 35.82 - .222y8
   y4c = 98000x3/(y2y7 + 1000x3)
   if |y4c - y4| ≤ .001  goto 60 else goto 50
50 y4 = y4c
   goto 40
60 stop
```

i	a_i	i	a_i
1	0	8	5000
2	0	9	2000
3	85	10	93
4	90	11	95
5	3	12	12
6	.01	13	4
7	145	14	162

Table 6: Data for test problem no. 67.

No. 70:

i	c_i	$y_{i,obs}$
1	.1	.00189
2	1	.1038
3	2	.268
4	3	.506
5	4	.577
6	5	.604
7	6	.725
8	7	.898
9	8	.947
10	9	.845
11	10	.702
12	11	.528
13	12	.385
14	13	.257
15	14	.159
16	15	.0869
17	16	.0453
18	17	.01509
19	18	.00189

Table 7: Data for test problem no. 70.

No. 83:

i	a_i	i	a_i
1	85.334407	7	.0029955
2	.0056858	8	.0021813
3	.0006262	9	9.300961
4	.0022053	10	.0047026
5	80.51249	11	.0012547
6	.0071317	12	.0019085

Table 8: Data for test problem no. 83.

No. 84:

i	a_i	i	a_i
1	-24345	11	15711.36
2	-8720288.849	12	-155011.1084
3	150512.5253	13	4360.53352
4	-156.6950325	14	12.9492344
5	476470.3222	15	10236.884
6	729482.8271	16	13176.786
7	-145421.402	17	-326669.5104
8	2931.1506	18	7390.68412
9	-40.427932	19	-27.8986976
10	5106.192	20	16643.076
		21	30988.146

Table 9: Data for test problem no. 84.

No. 85:

Let $y_i = y_i(x)$, $c_i = c_i(x)$.

$$y_1 = x_2 + x_3 + 41.6$$
$$c_1 = .024x_4 - 4.62$$
$$y_2 = 12.5/c_1 + 12$$
$$c_2 = .0003535x_1^2 + .5311x_1 + .08705y_2x_1$$
$$c_3 = .052x_1 + 78 + .002377y_2x_1$$
$$y_3 = c_2/c_3$$
$$y_4 = 19y_3$$
$$c_4 = .04782(x_1 - y_3) + .1956(x_1 - y_3)^2/x_2 + .6376y_4 + 1.594y_3$$
$$c_5 = 100x_2$$
$$c_6 = x_1 - y_3 - y_4$$
$$c_7 = .95 - c_4/c_5$$
$$y_5 = c_6c_7$$
$$y_6 = x_1 - y_5 - y_4 - y_3$$
$$c_8 = (y_5 + y_4).995$$
$$y_7 = c_8/y_1$$
$$y_8 = c_8/3798$$
$$c_9 = y_7 - .0663y_7/y_8 - .3153$$
$$y_9 = 96.82/c_9 + .321y_1$$
$$y_{10} = 1.29y_5 + 1.258y_4 + 2.29y_3 + 1.71y_6$$
$$y_{11} = 1.71x_1 - .452y_4 + .58y_3$$
$$c_{10} = 12.3/752.3$$
$$c_{11} = (1.75y_2)(.995x_1)$$
$$c_{12} = .995y_{10} + 1998$$
$$y_{12} = c_{10}x_1 + c_{11}/c_{12}$$
$$y_{13} = c_{12} - 1.75y_2$$

$$y_{14} = 3623 + 64.4x_2 + 58.4x_3 + 146312/(y_9 + x_5)$$

$$c_{13} = .995y_{10} + 60.8x_2 + 48x_4 - .1121y_{14} - 5095$$

$$y_{15} = y_{13}/c_{13}$$

$$y_{16} = 148000 - 331000y_{15} + 40y_{13} - 61y_{15}y_{13}$$

$$c_{14} = 2324y_{10} - 28740000y_2$$

$$y_{17} = 14130000 - 1328y_{10} - 531y_{11} + c_{14}/c_{12}$$

$$c_{15} = y_{13}/y_{15} - y_{13}/.52$$

$$c_{16} = 1.104 - .72y_{15}$$

$$c_{17} = y_9 + x_5$$

i	a_i	b_i
2	17.505	1053.6667
3	11.275	35.03
4	214.228	665.585
5	7.458	584.463
6	.961	265.916
7	1.612	7.046
8	.146	.222
9	107.99	273.366
10	922.693	1286.105
11	926.832	1444.046
12	18.766	537.141
13	1072.163	3247.039
14	8961.448	26844.086
15	.063	.386
16	71084.33	140000
17	2802713	12146108

Table 10: Data for test problem no. 85.

No. 86, 117:

j	1	2	3	4	5
e_j	-15	-27	-36	-18	-12
c_{1j}	30	-20	-10	32	-10
c_{2j}	-20	39	-6	-31	32
c_{3j}	-10	-6	10	-6	-10
c_{4j}	32	-31	-6	39	-20
c_{5j}	-10	32	-10	-20	30
d_j	4	8	10	6	2
a_{1j}	-16	2	0	1	0
a_{2j}	0	-2	0	4	2
a_{3j}	-3.5	0	2	0	0
a_{4j}	0	-2	0	-4	-1
a_{5j}	0	-9	-2	1	-2.8
a_{6j}	2	0	-4	0	0
a_{7j}	-1	-1	-1	-1	-1
a_{8j}	-1	-2	-3	-2	-1
a_{9j}	1	2	3	4	5
a_{10j}	1	1	1	1	1
b_j	-40	-2	-.25	-4	-4
b_{5+j}	-1	-40	-60	5	1

Table 11: Data for test problem no. 86, 117.

No. 88 - 92:

n	x*					
2	1.074319	-.4566137				
3	1.074319	-.4566137	.30E-10			
4	.708479	.24E-4	.807600	-.456614		
5	.701893	.22E-11	.813331	.456614	.90E-11	
6	.494144	-.10E-4	.614951	-.24E-5	.729259	-.456613

Table 12: Solution vectors for test problems no. 88 to 92.

n	r(x*)	e(x*)	$\lambda^*_{max}/\lambda^*_{min}$
2	.36E-11	.13E-6	.10E1
3	.73E-11	.94E-7	.20E2
4	.0	.14E-3	.65E8
5	.36E-11	.13E-6	.11E9
6	.0	.16E-2	.43E7

Table 13: Constraint violations, norm of Kuhn-Tucker-vector, and condition number for test problems no. 88 to 92.

No. 95 - 98:

i	b(95)	b(96)	b(97)	b(98)
1	4.97	4.97	32.97	32.97
2	-1.88	-1.88	25.12	25.12
3	-29.08	-69.08	-29.08	-124.08
4	-78.02	-118.02	-78.02	-173.02

Table 14: Data for test problems no. 95 to 92.

No. 99:

$b = 32$

i	a_i	t_i
1	0	0
2	50	25
3	50	50
4	75	100
5	75	150
6	75	200
7	100	290
8	100	380

Table 15: Data for test problem no. 99.

No. 101 - 103:

i	$x_i^*(101)$	$x_i^*(102)$	$x_i^*(103)$
1	2.856159	3.896253	4.394105
2	.6108230	.8093588	.8544687
3	2.150813	2.664386	2.843230
4	4.712874	4.300913	3.399979
5	.9994875	.8535549	.7229261
6	1.347508	1.095287	.8704064
7	.03165277	.02731046	.02463883

Table 16: Solution vectors for test problems no. 101 to 103.

No. 105:

i	y_i	i	y_i
1	95	168-175	175
2	105	176-181	180
3-6	110	182-187	185
7-10	115	188-194	190
11-25	120	195-198	195
26-40	125	199-201	200
41-55	130	202-204	205
56-68	135	205-212	210
69-89	140	213	215
90-101	145	214-219	220
102-118	150	220-224	230
119-122	155	225	235
123-142	160	226-232	240
143-150	165	233	245
151-167	170	234-235	250

Table 17: Data for test problem no. 105.

No. 109:

i	x_i^*	i	x_i^*	i	x_i^*
1	674.8881	4	-.3711526	7	201.465
2	1134.170	5	252.0000	8	426.661
3	.1335691	6	252.0000	9	368.494

Table 18: Solution vector for test problem no. 109

No. 111, 112:

j	c_j	j	c_j
1	-6.089	6	-14.986
2	-17.164	7	-24.100
3	-34.054	8	-10.708
4	-5.914	9	-26.662
5	-24.721	10	-22.179

Table 19: Data for test problems no. 111 and 112.

No. 114:

Bounds for test problem no. 114:

$$
\begin{array}{rcccl}
.00001 & \le & x_1 & \le & 2000 \\
.00001 & \le & x_2 & \le & 16000 \\
.00001 & \le & x_3 & \le & 120 \\
.00001 & \le & x_4 & \le & 5000 \\
.00001 & \le & x_5 & \le & 2000 \\
85 & \le & x_6 & \le & 93 \\
90 & \le & x_7 & \le & 95 \\
3 & \le & x_8 & \le & 12 \\
1.2 & \le & x_9 & \le & 4 \\
145 & \le & x_{10} & \le & 162
\end{array}
$$

No. 116:

Bounds for test problem no. 116:

$$.1 \le x_1 \le 1 \qquad .1 \le x_7 \le 1000$$
$$.1 \le x_2 \le 1 \qquad .1 \le x_8 \le 1000$$
$$.1 \le x_3 \le 1 \qquad 500 \le x_9 \le 1000$$
$$.0001 \le x_4 \le .1 \qquad .1 \le x_{10} \le 500$$
$$.1 \le x_5 \le .9 \qquad 1 \le x_{11} \le 150$$
$$.1 \le x_6 \le .9 \qquad .0001 \le x_{12} \le 150$$
$$.0001 \le x_{13} \le 150$$

No. 119:

j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
a_{1j}	1	0	0	1	0	0	1	1	0	0	0	0	0	0	0	1
a_{2j}	0	1	1	0	0	0	1	0	0	1	0	0	0	0	0	0
a_{3j}	0	0	1	0	0	0	1	0	1	1	0	0	0	1	0	0
a_{4j}	0	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0
a_{5j}	0	0	0	0	1	1	0	0	0	1	0	1	0	0	0	1
a_{6j}	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0
a_{7j}	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0	0
a_{8j}	0	0	0	0	0	0	0	1	0	1	0	0	0	0	1	0
a_{9j}	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1
a_{10j}	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
a_{11j}	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
a_{12j}	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0
a_{13j}	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
a_{14j}	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
a_{15j}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
a_{16j}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Table 20: Data for test problem no. 119.

j	b_{1j}	b_{2j}	b_{3j}	b_{4j}	b_{5j}	b_{6j}	b_{7j}	b_{8j}	c_j
1	.22	-1.46	1.29	-1.10	.0	.0	1.12	.0	2.5
2	.20	.0	-.89	-1.06	.0	-1.72	.0	.45	1.1
3	.19	-1.30	.0	.95	.0	-.33	.0	.26	-3.1
4	.25	1.82	.0	-.54	-1.43	.0	.31	-1.10	-3.5
5	.15	-1.15	-1.16	.0	1.51	1.62	.0	.58	1.3
6	.11	.0	-.96	-1.78	.59	1.24	.0	.0	2.1
7	.12	.80	.0	-.41	-.33	.21	1.12	-1.03	2.3
8	.13	.0	-.49	.0	-.43	-.26	.0	.10	-1.5
9	1.00	.0	.0	.0	.0	.0	-.36	.0	
10	.0	1.00	.0	.0	.0	.0	.0	.0	
11	.0	.0	1.00	.0	.0	.0	.0	.0	
12	.0	.0	.0	1.00	.0	.0	.0	.0	
13	.0	.0	.0	.0	1.00	.0	.0	.0	
14	.0	.0	.0	.0	.0	1.00	.0	.0	
15	.0	.0	.0	.0	.0	.0	1.00	.0	
16	.0	.0	.0	.0	.0	.0	.0	1.00	

Table 21: Data for test problem no. 119.

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